MATH 2025, fall 2009 Quiz 2 Name___________________

Please show all work, the correct arguments counts for half of the points! *(Please print)*

1[10P]) Which of the following sets is a vector space and which are not:

a) $\{(x, y) \in \mathbb{R}^2 \mid 2x + y^2 = 0\}$.

b) $\{(x, y, z) \in \mathbb{R}^n \mid 2x - 3y + z - 1 = 0\}$.

c) $\{f \in C^1((-1, 1)) \mid f'(0) = 0\}$.

d) $\{f \in C([0, 1]) \mid \int_0^1 f(t)\, dt = 0\}$.

e) The polynomials of degree at most 4, $\{a_4x^4 + a_3x^3 + a_2x^2 + a_1x + a_0 \mid a_0, a_1, a_2, a_3, a_4 \in \mathbb{R}\}$.

2[10P]) Which of the following maps are linear and which are not.

a) $T : \mathbb{R}^3 \to \mathbb{R}^4$,

$$T\left(\begin{pmatrix} x \\ y \\ z \end{pmatrix}\right) = \begin{pmatrix} 2x + 3y - z \\ 4x + yz \\ 3x - 3y + 5z \\ 3z \end{pmatrix}.$$

b) $T : C^1(\mathbb{R}) \to \mathbb{R}$, $T(f) = \int_0^1 f(t)\, dt$.

c) $T : C^1((0, 1)) \to C^0((0, 1))$, $T(f) = ff'$.

d) $T : C^0(\mathbb{R}) \to \mathbb{R}$, $T(f) = f(1)$.

e) For $n \in \mathbb{N}$ let $W_n = \left\{ \sum_{k=0}^{2^n-1} a_k^n \varphi_k^{(n)} \mid a_0, \ldots, a_{2^n-1} \in \mathbb{R} \right\}$ and $T : W_n \to W_{(n-1)}$ the Wavelet transform

$$T\Big(\sum_{k=0}^{2^n-1} a_k^{(n)} \varphi_k^{(n)}\Big) = \sum_{k=0}^{2^{n-1}-1} a_k^{(n-1)} \varphi_k^{(n)}, \quad a_k^{n-1} = \frac{a_{2k}^n + a_{2k+1}^n}{2} \text{ the average}.$$

MATH 2025, fall 2009 Quiz 2 Name Solutions

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1[10P]) Which of the following sets is a vector space and which are not:

a) $\{(x, y) \in \mathbb{R}^2 \mid 2x + y^2 = 0\}$. Not a vector space because of the y^2.

b) $\{(x, y, z) \in \mathbb{R}^n \mid 2x - 3y + z - 1 = 0\}$. Not a vector space because of the -1. $(0,0,0)$ is not in the set

c) $\{f \in C^1((-1, 1)) \mid f'(0) = 0\}$. Is a vector space: $(f+g)'(0) = f'(0) + g'(0) = 0$. $(rf)'(0) = r f'(0) = 0$

d) $\{f \in C([0, 1]) \mid \int_0^1 f(t)\, dt = 0\}$. Is a vector space, because $\int_0^1$ is linear

e) The polynomials of degree at most 4, $\{a_4x^4 + a_3x^3 + a_2x^2 + a_1x + a_0 \mid a_0, a_1, a_2, a_3, a_4 \in \mathbb{R}\}$. Is a vector space.

2[10P]) Which of the following maps are linear and which are not.

a) $T : \mathbb{R}^3 \to \mathbb{R}^4$,

$$T\left(\begin{pmatrix} x \\ y \\ z \end{pmatrix}\right) = \begin{pmatrix} 2x + 3y - z \\ 4x + yz \\ 3x - 3y + 5z \\ 3z \end{pmatrix}.$$

not linear

b) $T : C^1(\mathbb{R}) \to \mathbb{R}$, $T(f) = \int_0^1 f(t)\, dt$. Is linear $\int_0^1 rf + sg\, dt = r\int_0^1 f\,dt + s\int_0^1 g\, dt$

c) $T : C^1((0, 1)) \to C^0((0, 1))$, $T(f) = ff'$.

not linear $T(rf) = r^2 ff' = r^2 T(f) \neq rT(f)$ if $r \neq 1, 0$

d) $T : C^0(\mathbb{R}) \to \mathbb{R}$, $T(f) = f(1)$.

Linear

e) For $n \in \mathbb{N}$ let $W_n = \left\{ \sum_{k=0}^{2^n-1} a_k^n \varphi_k^{(n)} \mid a_0, \ldots, a_{2^n-1} \in \mathbb{R} \right\}$ and $T : W_n \to W_{(n-1)}$ the Wavelet transform

$$T\left(\sum_{k=0}^{2^n-1} a_k^{(n)} \varphi_k^{(n)}\right) = \sum_{k=0}^{2^{n-1}-1} a_k^{(n-1)} \varphi_k^{(n)}, \quad a_k^{n-1} = \frac{a_{2k}^n + a_{2k+1}^n}{2} \text{ the average}.$$

linear.