MATH 2065-4, Fall 2010 Name: Solutions

Test 3, Thursday, Nov. 18, 2010. For partial credit, show all your work!

1[12P]) For each of the following differential equations determine if it is linear (Y=linear) or not (N=not):

$L(y)$	Y	N
$y'' + y' + y = \sin(t)$	X	
$y'' + y'y = t$		X
$y'' + \cos(y) = \cos(t)$		X
$y'' + y' + \cos(t) = y$	X	

2[6P]) Show that $y(t) = t^{1/2}$ is a solution to the differential equation $4t^2y'' + y = 0$.

$y = t^{1/2}$

$y' = \frac{1}{2} t^{-1/2}$

$y'' = -\frac{1}{4} t^{-3/2}$

$4t^2 y'' + y = -t^{1/2} + t^{1/2} = 0$

3[7]) For the linear operator $L = D^2 + 4D + 4$ determine $L(te^{-t}) =$

$y = te^{-t}$

$y' = -te^{-t} + e^{-t}$

$y'' = te^{-t} - 2e^{-t}$

$L(te^{-t}) = te^{-t} - 2e^{-t} - 4te^{-t} + 4e^{-t} + 4te^{-t} = te^{-t} + 2e^{-t}$

4[45P]) Find the general solution to each of the following differential equations:

a) $y'' - 3y' + 2y = e^t$. **Solution** $y(t) =$ $c_1 e^t + c_2 e^{2t} - te^t$

$s^2 - 3s + 2 = (s-1)(s-2)$

$y_h = c_1 e^t + c_2 e^{2t}$

$y_p \Leftrightarrow \frac{1}{(s-1)^2(s-2)} = \frac{-1}{(s-1)^2} + \frac{1}{(s-1)(s-2)}$

$y_p = -te^t$

b) $y'' - 4y' + 4y = e^t$. **Solution** $y(t) =$ $c_1 e^{2t} + c_2 te^{2t} + e^t$

$s^2 - 4s + 4 = (s-2)^2$

$y_p = Ae^t, y_p' = Ae^t, y_p'' = Ae^t$

$L(Ae^t) = Ae^t = e^t. \ A = 1$

c) $y'' - 4y' + 5y = 1$. **Solution** $y(t) = e^{2t}(c_1 \cos(t) + c_2 \sin(t)) + \frac{1}{5}$

$s^2 - 4s + 5 = (s-2)^2 + 1$

5[15P]) Find the general solution to the differential equation $y'' + y' - 2y = te^t$.

Solution: $y(t) =$

$s^2 + s - 2 = (s+2)(s-1)$ $\quad y_h(t) = c_1 e^{-2t} + c_2 e^t$

$y_p(t)$: write $y_p = c_1(t) e^{-2t} + c_2(t) e^t$. Then

$c_1' e^{-2t} + c_2' e^t = 0$

$-2c_1' e^{-2t} + c_2' e^t = te^t$

$3c_2' e^t = te^t$ or $c_2' = \frac{1}{3}t$

$c_2 = \frac{1}{3}\int t\,dt = \frac{1}{6}t^2$

$3c_1' e^{-2t} = -te^t$

$c_1' = -\frac{1}{3} t e^{3t}$

$c_1 = -\frac{1}{3}\int t e^{3t} dt$

$= -\frac{1}{9} t e^{3t} + \frac{1}{9}\int e^{3t} dt$

$= -\frac{1}{9} t e^{3t} + \frac{1}{27} e^{3t}$.

$y_p = -\frac{1}{9} t e^t + \frac{1}{27} e^t + \frac{1}{6} t^2 e^t$

$y = c_1 e^{-2t} + c_2 e^t +$
$+\left(\frac{1}{6}t^2 - \frac{1}{9}t + \frac{1}{27}\right)e^t$.

6[15P]) Solve the initial value problem $t^2 y'' + ty' - 4y = 0$, $y(1) = 1$, $y'(1) = 1$.

Solution: $y(t) = \frac{1}{4}(3t^2 - t^{-2})$

Euler equation: $Y''(x) - 4Y(x) = 0$, $Y(x) = c_1 e^{2x} + c_2 e^{-2x}$

$y(t) = c_1 t^2 + c_2 t^{-2}$

$y'(t) = 2c_1 t^2 - 2c_2 t^{-2}$

$t=1$: $c_1 + c_2 = 1$

$2c_1 - 2c_2 = 1$

$c_1 = 3/4, \; c_2 = 1/4$