Math 2025, Problems

I) Write the 2×2-matrix for the following functions:

(1) $f = 2\varphi \otimes \psi$. **Answer:** $\begin{pmatrix} 2 & 2 \\ 2 & -2 \end{pmatrix}$

(2) $f = \varphi_1^1 \otimes \varphi - \psi \otimes \varphi_0^1$. **Answer:**

$$\begin{pmatrix} 0 & 0 \\ 1 & 1 \end{pmatrix} - \begin{pmatrix} 1 & 0 \\ -1 & 0 \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 2 & 1 \end{pmatrix}$$

(3) $f = -3\varphi \otimes \varphi_1^1$ **Answer** $\begin{pmatrix} 0 & -3 \\ 0 & -3 \end{pmatrix}$

(4) $f = 10\cos(\pi\cdot) \otimes \psi$. **Answer:** $\begin{pmatrix} 10 & -10 \\ 0 & 0 \end{pmatrix}$

Recall that the 2×2 matrix corresponding to a function $f(x, y)$ is given by

$$\begin{pmatrix} f(0,0) & f(0,1/2) \\ f(1/2,0) & f(1/2,1/2) \end{pmatrix}$$

II) What is the Haar wavelet transform of the following 2×2 matricies:

(1) $\begin{pmatrix} 2 & 4 \\ 0 & 2 \end{pmatrix}$. **Solution:** Recall that you have to apply the one dimensional Haar wavelet transform to the rows and the columns. **It does not matter in which order you do it!**

$$\begin{aligned} \begin{pmatrix} 2 & 4 \\ 0 & 2 \end{pmatrix} &\to \begin{pmatrix} \frac{2+4}{2} & \frac{2-4}{2} \\ \frac{0+2}{2} & \frac{0-2}{2} \end{pmatrix} = \begin{pmatrix} 3 & -1 \\ 1 & -1 \end{pmatrix} \\ &\to \begin{pmatrix} \frac{3+1}{2} & \frac{-1-1}{2} \\ \frac{3-1}{2} & \frac{1+1}{2} \end{pmatrix} = \begin{pmatrix} 2 & -1 \\ 1 & 0 \end{pmatrix} \end{aligned}$$

(2) $\begin{pmatrix} 1 & 3 \\ -1 & 5 \end{pmatrix}$. **Solution:**

$$\begin{pmatrix} 1 & 3 \\ -1 & 5 \end{pmatrix} \to \begin{pmatrix} \frac{1+3}{2} & \frac{1-3}{2} \\ \frac{-1+5}{2} & \frac{-1-5}{2} \end{pmatrix} = \begin{pmatrix} 2 & -1 \\ 2 & -3 \end{pmatrix}$$
$$\to \begin{pmatrix} \frac{2+2}{2} & \frac{-1-3}{2} \\ \frac{2-2}{2} & \frac{-1+3}{2} \end{pmatrix} = \begin{pmatrix} 2 & -2 \\ 0 & 1 \end{pmatrix}$$

(3) $\begin{pmatrix} 5 & 0 \\ 2 & 1 \end{pmatrix}$

(4) $\begin{pmatrix} 3 & 1 \\ 2 & -2 \end{pmatrix}$

(5) $\begin{pmatrix} 0 & 4 \\ 6 & 8 \end{pmatrix}$

III) Apply the inverse Haar wavelet transform to the following matricies:

(1) $\begin{pmatrix} 1 & 2 \\ -1 & 2 \end{pmatrix}$. **Solution:** Recall you have to apply the inverse one-dimensional transform to the columns and the rows. **It does not matter in which order you do that**

$$\begin{pmatrix} 1 & 2 \\ -1 & 2 \end{pmatrix} \to \begin{pmatrix} 1-1 & 2+2 \\ 1+1 & 2-2 \end{pmatrix} = \begin{pmatrix} 0 & 4 \\ 2 & 0 \end{pmatrix}$$
$$\to \begin{pmatrix} 0+4 & 0-4 \\ 2+0 & 2-0 \end{pmatrix} = \begin{pmatrix} 4 & -4 \\ 2 & 2 \end{pmatrix}$$

(2) $\begin{pmatrix} 1 & 0 \\ 1 & 5 \end{pmatrix}$ **Solution:**

$$\begin{pmatrix} 1 & 0 \\ 1 & 5 \end{pmatrix} \to \begin{pmatrix} 1+0 & 1-0 \\ 1+5 & 1-5 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 6 & -4 \end{pmatrix}$$
$$\to \begin{pmatrix} 1+6 & 1-4 \\ 1-6 & 1+4 \end{pmatrix} = \begin{pmatrix} 7 & -3 \\ -5 & 5 \end{pmatrix}$$

(3) $\begin{pmatrix} 3 & -1 \\ 1 & 1 \end{pmatrix}$

(4) $\begin{pmatrix} 0 & 1 \\ -2 & -2 \end{pmatrix}$

(5) $\begin{pmatrix} 0 & -4 \\ 3 & 8 \end{pmatrix}$