Math 1550-22, Test # 1. Fall 2008 Name: ____________

No calculator, no notes, no book.

1[15P]) Let $f(x) = \frac{x^2+1}{x-1}$.

a) What is the domain of the function $f(x)$? The domain is: $x \neq 1$

b) Find the average rate of change over the interval $[2, 3]$. The AROC is : ____

$$\frac{f(3)-f(2)}{3-2} = \frac{\frac{10}{2} - 5}{1} = 0$$

c) What is the instantaneous rate of change at $x = 2$? -1

$$f'(x)\Big|_{x=2} = \frac{2x(x-1)-(x^2+1)}{(x-1)^2}\Big|_{x=2} = \frac{x^2-2x-1}{(x-1)^2}\Big|_{x=2} = \frac{4-4-1}{1^2} = -1$$

d) $\lim_{x\to 1+} f(x) = +\infty$. (The limit does not exist)

e) $\lim_{x\to 1-} f(x) = -\infty$. (-11-)

2[20P]) Evaluate the following limits. **Show your work, justify your answer!**:

a) $\lim_{x\to 4} \frac{\sqrt{x}-2}{x-4} =$ $1/4$.

$$\frac{\sqrt{x}-2}{x-4} = \frac{x-4}{x-4}\cdot\frac{1}{\sqrt{x}+2} = \frac{1}{\sqrt{x}+2} \xrightarrow{x\to 4} \frac{1}{4}$$

b) $\lim_{x\to 0} \frac{\cos(x)\sin(3x)}{5x} =$ $3/5$.

$$= \left(\lim_{x\to 0} \cos x\right)\left(\lim_{x\to 0} \frac{\sin(3x)}{5x}\right) = 1\cdot\frac{3}{5}\lim_{x\to 0}\frac{\sin 3x}{3x}$$

$$= \frac{3}{5}$$

c) $\lim_{x\to 0} \frac{\tan(2x)}{\tan(3x)} =$ $2/3$.

$\frac{2}{3} \frac{\sin(2x)}{2x} \cdot \frac{3x}{\sin(3x)} \cdot \frac{\cos(3x)}{\cos(2x)}$

$\to \frac{2}{3} \cdot 1 \cdot 1 \cdot \frac{1}{1} = 2/3$

d) $\lim_{h\to 0} \frac{(4+h)^2-16}{h} =$ 8.

$\frac{(4+h)^2-16}{h} = \frac{16+8h+h^2-16}{h} = 8+h \xrightarrow[h\to 0]{} 8$

$\lim_{x\to 1} \frac{x^2-9x+8}{x^2-1} =$ _____.

$\frac{x^2-9x+8}{x^2-1} = \frac{x-8}{x+1} \xrightarrow{x\to 1} \frac{-7}{2}$

$x^2-1 = (x-1)(x+1)$

$x^2-9x+8 = (x-1)(x-8)$

3[5P]) For which number(s) c is the function $f(x) = \begin{cases} x^2+c & \text{if } x<2 \\ \sqrt{x^2-3}-c & \text{if } x\geq 2 \end{cases}$ continuous ?

$c =$ $-3/2$.

insert $x=2$: $4+c = 1-c$ or $2c = -3$, $c = -3/2$

4[10P]) Let $f(x) = \begin{cases} x^2+1 & \text{if } x<1 \\ 3 & \text{if } x=1 \\ \sqrt{x^2-1}+2 & \text{if } x\in(1,2] \\ x+3 & \text{if } x\in(2,\infty) \end{cases}$.

a) Evaluate $\lim_{x\to 1-} f(x) =$ 2 and $\lim_{x\to 1+} f(x) =$ 2. Is $f(x)$ continuous at the point $x=1$? No, because $f(1) = 3$.

b) Is $f(x)$ continuous at the point $x=2$? no. because

$\lim_{x\to 2-} f(x) = \sqrt{3}+2 \neq \lim_{x\to 2+} f(x) = 5.$

5[10P]) Use the **definition** of the derivative to find the derivative of the function $f(x) = \sqrt{x+2}$. **Show your work!**
$f'(x) =$ $1/2\sqrt{x+2}$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{\sqrt{x+2+h}-\sqrt{x+2}}{h} =$$

$$= \lim_{h\to 0} \frac{(x+2+h)-(x+2)}{h} \cdot \frac{1}{\sqrt{x+2+h}+\sqrt{x+2}} = \frac{1}{2\sqrt{x+2}}$$

6[10P]) Consider the functions $f(x)$ and $g(x)$, for which $f(0) = 3$, $g(0) = 2$, $f'(0) = 1$, and $g'(0) = 2$. Let $h(x) = \frac{f(x)}{g(x)}$. Find $h'(0) =$ -1 .

Use the quotient rule

$$\left(\frac{f}{g}\right)'(x) = \frac{f'(x)g(x) - f(x)g'(x)}{g(x)^2} : \quad \frac{1\cdot 2 - 3\cdot 2}{2^2} = \frac{2-6}{4} = -1$$

7[20P]) Find the following derivatives:
a) $f(x) = 2x^3 + x + x^{-2}$, $f'(x) =$ $6x^2+1-2x^{-3}$

Use the power rule $\frac{d}{dx}x^k = kx^{k-1}$

b) $f(t) = te^t$, $f'(t)|_{t=0} =$ 1 .

Use the product rule and $e^0 = 1$.

$$f'(t)|_{t=0} = (e^t + te^t)|_{t=0} = 1$$

c) $g(z) = \left(\frac{z^2-4}{z-1}\right)\left(\frac{z^2-1}{z+2}\right)$, $g'(z) =$ $2z-1$.

Simplify

$$\left(\frac{z^2-4}{z-1}\right)\cdot\left(\frac{z^2-1}{z+2}\right) = \frac{(z+2)(z-2)(z-1)(z+1)}{(z-1)(z+2)}$$

$$= (z-2)(z+1) = z^2 - z - 2$$

d) $f(x) = \dfrac{x^2 + 3x + x^{-1}}{x}$, $f'(x) =$ $1 - 2x^{-3}$

simplify: $f(x) = x + 3 + x^{-2}$ and then use the power rule

8[10P]) The hight $s(t)$ of an object tossed vertically in the air is given by $s(t) = s_0 + v_0 t - \frac{1}{2} g t^2$, $g = 32$ ft/sec^2.

a) Explain the meaning of s_0, v_0, and g.

S_0 is the initial hight (the hight at time $t=0$),
V_0 is the initial velocity
g is the gravitational constant.

b) If $s_0 = 2$ft, $v_0 = 320$ft/sec. What is the maximum hight and when does it reach that hight?
Maximum hight: ________ and the time ________

maximum hight: $\frac{ds}{dt}(t) = V(t) = 0$

$S(t) = 2 + 320t - 16t^2$

$V(t) = 320 - 36t = 36(10 - t)$

If $V(t) = 0$ then $t = 10$. Hence the max hight is $S(10) = 2 + 3200 - 1600 = 1602$ ft.

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c) What is the instantaneous rate of change at $x = 2$? ______

d) $\lim_{x\to 1+} f(x) =$ ______ .

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2[20P]) Evaluate the following limits. **Show your work, justify your answer!**:

a) $\lim_{x\to 4} \dfrac{\sqrt{x}-2}{x-4} =$ ______ .

b) $\lim_{x\to 0} \dfrac{\cos(x)\sin(3x)}{5x} =$ ______ .

c) $\lim\limits_{x\to 0} \dfrac{\tan(2x)}{\tan(3x)} =$ __________ .

d) $\lim\limits_{h\to 0} \dfrac{(4+h)^2-16}{h} =$ __________ .

$\lim\limits_{x\to 1} \dfrac{x^2-9x+8}{x^2-1} =$ __________ .

3[5P]) For which number(s) c is the function $f(x) = \begin{cases} x^2+c & \text{if } x<2 \\ \sqrt{x^2-3}-c & \text{if } x \geq 2 \end{cases}$ continuous ?

$c =$ __________ .

4[10P]) Let $f(x) = \begin{cases} x^2+1 & \text{if } x<1 \\ 3 & \text{if } x=1 \\ \sqrt{x^2-1}+2 & \text{if } x\in(1,2] \\ x+3 & \text{if } x\in(2,\infty) \end{cases}$.

a) Evaluate $\lim\limits_{x\to 1-} f(x) =$ __________ and $\lim\limits_{x\to 1+} f(x) =$ __________ . Is $f(x)$ continuous at the point $x=1$? __________ .

b) Is $f(x)$ continuous at the point $x=2$? __________ .

5[10P]) Use the **definition** of the derivative to find the derivative of the function $f(x) = \sqrt{x+2}$.
Show your work!
$f'(x) =$ ____________.

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b) $f(t) = te^t$, $f'(t)|_{t=0} =$ ____________.

c) $g(z) = \left(\frac{z^2-4}{z-1}\right)\left(\frac{z^2-1}{z+2}\right)$, $g'(z) =$ ____________.

d) $f(x) = \dfrac{x^2 + 3x + x^{-1}}{x}$, $f'(x) =$ ____________ .

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Maximum hight: ____________ and the time ____________