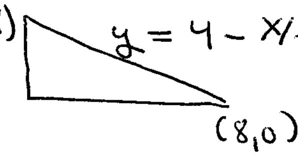
Math 208-1, Test # 3. Fall 2007 **Name:** ____________________

1[14P]) Find the centroid of the triangle with vertices at $(0,0)$, $(0,4)$, and $(8,0)$.

$\bar{x} =$ $8/3$ $\bar{y} =$ $4/3$

The triangle



$m = \text{area} = 16$

$$\bar{x} = \frac{1}{16}\int_0^8 \int_0^{4-\frac{x}{2}} x\,dy\,dx = 8/3$$

$$\bar{y} = \frac{1}{16}\int_0^8 \int_0^{4-\frac{x}{2}} y\,dy\,dx = \frac{1}{16}\cdot\frac{1}{2}\int_0^8 \left(4-\frac{x}{2}\right)^2 dx = 4/3$$

2[9P]) Set up the integral for the surface area $A(S)$ of the part of the circular paraboloid $z = x^2+y^2$ that lies inside the cylinder $x^2+y^2=1$. $A(S) =$ $\displaystyle\iint_{x^2+y^2\le 1} \sqrt{1+4(x^2+y^2)}\,dx\,dy$

$$A(S) = \iint_D \sqrt{1+\|\nabla z\|^2}\,dA$$

$$= \iint \sqrt{1+4(x^2+y^2)}\,dA$$

$$= \int_0^{2\pi}\int_0^1 \sqrt{1+4r^2}\,r\,dr\,d\theta$$

3[27P]) Evaluate the triple integrals. You might want to consider what coordinates is best to use!

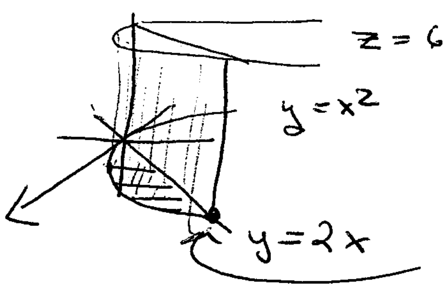
a) $\displaystyle\int_0^1 \int_0^{\cancel{x}\,z} \int_0^{x+z} 6z\,dy\,dx\,dz =$ $9/4$

$$\int_0^{x+z} 6z\,dy = 6xz + 6z^2$$

$$\int_0^z 6xz + 6z^2\,dx = 3z^3 + 6z^3 = 9z^3$$

$$\int_0^1 9z^3\,dz = \frac{9}{4}$$

b) Let E be bounded by the parabolic cylinder $y = x^2$ and the planes $z = 6$, $y = 2x$, and $z = 0$. Evaluate $\iiint_E x\,dV =$ $\underline{\quad 8 \quad}$



$$\int_0^2 \int_{x^2}^{2x} \int_0^6 x\, dz = 6\int_0^2 x(2x - x^2)\,dx$$

$$= 6\left[\frac{2}{3}x^3 - \frac{x^4}{4}\right]_0^2 = 8$$

$x^2 = 2x,\ x \neq 0,\ x = 2$

c) $\displaystyle\int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}}\int_{x^2+y^2}^{2-x^2-y^2} (x^2+y^2)^{5/2}\,dzdydx =$ $\underline{\quad 8\pi/63 \quad}$

use cylindrical coordinates

$$\int_0^{2\pi}\int_0^1\int_{r^2}^{2-r^2} r^5 \cdot r\,dz\,dr\,d\theta = 2\pi\int_0^1 r^6(2 - r^2 - r^2)\,dr$$

$$= 2\pi \int_0^1 2r^6 - 2r^8\,dr = \frac{8\pi}{63}$$

4[12P]) Use spherical coordinates to evaluate the integral $\displaystyle\iiint_E \frac{e^{-(x^2+y^2+z^2)}}{\sqrt{x^2+y^2+z^2}} =$ $\underline{\quad \frac{2\pi(e-1)}{e} \quad}$ where E is the unit ball $x^2 + y^2 + z^2 \le 1$.

$$\int_0^{\pi}\int_0^{2\pi}\int_0^1 \frac{e^{-r^2}}{\cancel{r}} r^{\cancel{2}} \sin\phi\,dr\,d\theta\,d\phi = 2\pi\left[-\cos\phi\right]_0^{\pi}\left[-\frac{1}{2}e^{-r^2}\right]_0^1$$

$$= 2\pi \cdot 2 \cdot \left[\frac{1}{2}\left[-e^{-1} + 1\right]\right]$$

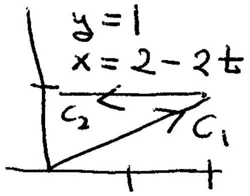
$$= 2\pi\left[1 - \frac{1}{e}\right]$$

5[6P]) Compute the gradient vector field of the following two functions:

A) $f(x,y) = \dfrac{1}{\sqrt{x^2+y^2}}$, $\nabla f(x,y) =$ $\underline{\frac{x}{r^3}}\,\mathbf{i} +$ $\underline{\frac{y}{r^3}}\,\mathbf{j}$. $\quad r = \sqrt{x^2+y^2}$

B) $f(x,y,z) = xye^{z^2} + \cos(xy)$, $\nabla f(x,y,z) = (\underline{ye^{z^2} - y\sin(xy)}, \underline{xe^{z^2} - x\sin(xy)}, \underline{2xyze^{z^2}})$.

6[12P]) Evaluate the line integral $\int_C 2x\,ds = \underline{2+\sqrt{5}}$ where C consist of the line segment from $(0,0)$ to $(2,1)$, followed by the line segment from $(2,1)$ to $(0,1)$.



$x = t,\ y = \frac{1}{2}t$ $\qquad ds = |\nabla \vec{r}|\,dt$

$\nabla \vec{r} = (1, \frac{1}{2}),\ |\nabla \vec{r}| = \frac{1}{2}\sqrt{5}$

$$\int_{C_1} x\,ds = \frac{\sqrt{5}}{2}\int_0^2 t\,dt = \frac{\sqrt{5}}{4}t^2\Big]_0^2 = \sqrt{5}$$

$$\int_{C_2} x\,ds = \int_0^1 (2-2t)\cdot 2\,dt = 2$$

7[8P]) Determine which of the following vector fields in conservative:

A) $\mathbf{F}(x,y) = (2xy, x^2+3y^2)$.

$\frac{\partial P}{\partial y} = 2x,\ \frac{\partial Q}{\partial x} = 2x$ conservative

B) $\mathbf{F}(x,y) = (x-y, xy)$.

$\frac{\partial P}{\partial y} = -1,\ \frac{\partial Q}{\partial x} = y.$ Not conservative

8[12P]) Evaluate the line integral $\int_C \mathbf{F}\cdot d\mathbf{r}$ $\underline{\quad 0 \quad}$, where $\mathbf{F}(x,y) = (2xy, x^2+1)$ and C is given by the vector function $\mathbf{r}(t) = (\cos(t), \sin(t))$, $0 \le t \le \pi$.

$\frac{\partial P}{\partial y} = 2x$

$\frac{\partial Q}{\partial x} = 2x$

So $\vec{F}$ is conservative

$\vec{F} = \nabla(x^2 y)$

$$\int_C F\cdot dr = x^2 y\Big]_{(1,0)}^{(-1,0)} = 0$$