Math 7370, Section I
Spring 2002
Lie groups and representations

Time: 12:10–1:30, Tuesday and Thursday in Lockett 132.
Instructor: Gestur Olafsson
Office: 322 Lockett
Office Hours: Tuesday, 3:00-4:00 and Thursday 11:00-12:00. You can also contact me by e-mail for other appointments.
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This is an introductory course on Lie Groups and representation theory. Though we will discuss Lie groups in general, our main discussion will focus on linear Lie groups. Those are the closed subgroups of the general linear group $\mathrm{GL}(n,\mathbb{R})$. This will give many of the most interesting examples and the concepts in this setting are more easily understood. Central examples include the linear groups $\mathrm{SO}(n)$ and $\mathrm{SL}(n,\mathbb{R})$. Topics shall include:

(1) Linear groups, basic definition and examples. The corresponding Lie algebra;
(2) Short review of differential geometry: Manifolds, vector fields, tangent space, differentiable mappings between manifolds and the deriviative;
(3) Definition of Lie groups and their Lie algebras;
(4) Lie subalgebras and Lie subgroups;
(5) Homomorphisms;
(6) The exponential map;
(7) Action of Lie groups on manifolds, homogeneous spaces; Typical examples are the action of the orthgonal group $\mathrm{SO}(n)$ on S^{n-1}, and the flag manifolds;
(8) Representations of Lie groups and Lie algebras. We discuss some natural examples like the connection between the natural representations given by the actions of $\mathrm{SO}(n)$ on S^{n-1} and $\mathbb{R}^n$ and spherical harmonics.
(9) (If there is time left) the Plancherel Theorem for compact Lie groups.

I expect that students have a basic understanding and knowledge in analysis and differential geometry (even if I will review most of what we will need).

Grades

I will organize a **problem hour** every second week or more often if necessary. There problems that I will hand out will be discussed. There will be three sets of home work that I will grade and one final take home test.

Books

The course will not be based on any special book. But here is a short list of interesting books, that are close to what we will do:

(1) T. Bröcker and T. tom Dieck: Representations of Compact Lie groups. (Springer) [Good for representations of compact Lie groups, the general theory of Lie group a litle too short]

(2) F. W. Warner: Foundations of Differentiable Manifolds and Lie Groups. (Springer) [Good for differential geometry and Lie groups. We will use this one a lot]

(3) V. S. Varadarajan: Lie Groups, Lie Algebras, and Their Representations. (2^{th}-edition, Springer) [Good for the general theory, but goes into too many subject for a class like ours]

(4) R. Goodman and N. R. Wallach: Representations and Invariants of the Classical Groups. (Cambridge) {Great]

(5) W. Fulton and J. Harris: Representation Theory, A First Course. (Springer). Has lots of examples.