Wavelets, Problems #2

Due, Friday Feb. 21

1) Suppose that f and g both have compact support, are integrable, and that g is smooth. Show that $f * g \in C_c^\infty(\mathbb{R}^n)$=the space of smooth compactly supported functions on $\mathbb{R}^n$.

2) Show that the space $\mathcal{S}(\mathbb{R}^n)$ is dense in $L^1(\mathbb{R}^n)$. (Hint: Approximate $f \in L^1(\mathbb{R}^n)$ by a compactly supported function g and then use convolution to make a smooth function out of g.)

3) Let $f \in L^1(\mathbb{R})$. Show that

$$\lim_{|x| \to \infty} \hat{f}(x) = 0.$$