Wavelets, Problems due Fr. April 11, 2003

1) Let $\varphi(x) = \text{sinc}(\pi x) = \frac{\sin(\pi x)}{\pi x}$. Find a sequence $\{h_n\} \in \ell^2(\mathbb{Z})$ such that

$$\varphi(x) = \sqrt{2} \sum_{n=-\infty}^{\infty} h_n \varphi(2x - n) \ .$$

(Hint: Find the Fourier transform of $\varphi(x)$ and $\varphi(2x-n)$. Then write the above equation in the form

$$\hat{\varphi}(\omega) = \hat{\varphi}(\omega/2) H(\omega) \ .$$

Then use Fourier series to expand $H(\omega)$.)