

PROBLEMS, MATH 2025

1) Let $V \subset \mathbb{R}^2$ be the line $V = \mathbb{R}(1, -1)$.

- a) Write the formula for the orthogonal projection $P: \mathbb{R}^2 \rightarrow V$
b) What is: a) $P(1, 1)$ b) $P(2, 1)$, c) $P(2, -2)$?

2) Let $W \subset \mathbb{R}^3$ be the plane

$$W = \{(x, y, z) \in \mathbb{R}^3 : x - 2y + z = 0\}.$$

a) Find the orthogonal projection $P: \mathbb{R}^3 \rightarrow W$.

- b) What is: i) $P(1, 1, 2)$, ii) $P(1, -3, 1)$, iii) $P(2, 1, 1)$.

3) Let $W \subset \mathbb{R}^3$ be the plane generated by the vectors $(1, 1, 1)$ and $(1, -1, 1)$.

a) Find the orthogonal projection $P: \mathbb{R}^3 \rightarrow W$.

- b) What is: i) $P(1, 1, 2)$, ii) $P(2, 0, 1)$.

4) Let W be the space of continuous functions on $[0, 1]$ generated by the constant function $x \mapsto 1$ and x . Thus

$W = \{a_0 + a_1 x : a_0, a_1 \in \mathbb{R}\}$. Find the orthogonal projection of the following functions onto W .

- a) $P(x^2)$, $P(e^x)$, $P(1 + x^2)$.

5) Let W be the space of piecewise continuous functions on $[0, 1]$ generated by $\chi_{[0, 1/2]}$ and $\chi_{[1/2, 1]}$. Find the orthogonal projection of the following functions onto W .

- i) $P(x)$, ii) $P(x^2)$, $P(\chi_{[0, 3/4]})$