Exercises for lecture 2

1) Determine which of the following maps are linear or not:

a) $T : \mathbb{R}^2 \to \mathbb{R}^3$, $T(x, y) = (x + 3y, 2xy - 1, x)$

b) $T : \mathbb{R}^2 \to \mathbb{R}^3$, $T(x, y) = (3x + 2y, x - y)$;

c) $T : \mathbb{R}^3 \to \mathbb{R}^3$, $T(x, y, z) = (x - z, x + zy, z)$;

d) $T : \mathbb{R}^3 \to \mathbb{R}^2$, $T(x, y, z) = (x + 4y - z, x + y - 1)$;

e) $T : C^1([0, 1)) \to C([0, 1))$, $T(f)(x) = \int f(t)t^2\, dt$;

f) $T : C^1([0, 1)) \to C([0, 1))$, $T(f)(t) = \int f(t)\, dt$;

g) $T : C^1(\mathbb{R}) \to C(\mathbb{R})$, $T(f) = f'f$;

h) $T : C([0, 1]) \to \mathbb{R}$, $T(f) = \int_0^1 f(t)^2\, dt$;

i) $T : C^\infty(\mathbb{R}) \to C^\infty(\mathbb{R})$, $T(f) = f'' + f + 2$;

2) Evaluate the given linear map T at the given point u, (or $f(t)$):

a) $T(x, y) = (x - 3y, 2x - y)$, $u = (2, -4)$;

b) $T(x, y, z) = (2x - y + 3z, 2x + 2y, -3x + 10y - z)$, $u = (1, -2, 3)$;

c) $T : C^1((-1, 1)) \to \mathbb{R}$, $T(f) = f'(0) + f(0)$, $f(t) = \cos(t)$;

d) $T : C([0, 1]) \to \mathbb{R}$, $T(f) = \int_0^1 f(t)t\, dt$, $f(t) = e^t$;

e) $T : \mathbb{R}^3 \to \mathbb{R}^2$,

$$T(x, y, z) = (x, y, z) \begin{pmatrix} 1 & -1 & 0 \\ 0 & -1 & 3 \\ 2 & 3 & -1 \end{pmatrix}.$$

f) $T(x, y) = (2x - y, 3x + y, 2y)$, $u = (1, -2)$;

3) Find the kernel of the following linear maps:

a) $T : \mathbb{R}^3 \to \mathbb{R}^3$, $T(x, y) = (2x + 3y, 2y - z, 2x + y + z)$

b) $T : \mathbb{R}^2 \to \mathbb{R}^2$, $T(x, y) = (x + 3y, 2x + y)$

c) $T : C^1(\mathbb{R}) \to C(\mathbb{R})$, $T(f) = f'$.