Math 2057-5 Quiz #1 (Fall 2005) **Name: Solution**

1) Let $f(x,y) = x\ln(3x+2y)$, $3x+2y>0$. Find the partial derivative f_{xy}.

Solution: $f_{xy}(x,y) = \dfrac{4y}{(3x+2y)^2}$.

First we have to find the partial derivative f_x, and then we have to differentiate f_x with respect to the variable y.

We have

$$f_x(x,y) = \ln(3x+2y) + x\frac{3}{3x+2y} = \ln(3x+2y) + \frac{3x}{3x+2y}\,.$$

Next we find the partial derivative of f_x with respect to y:

$$\begin{aligned}(f_x)_y(x,y) &= \frac{2}{3x+2y} - \frac{3x\cdot 2}{(3x+2y)^2}\\ &= \frac{2\cdot(3x+2y)-6x}{(3x+2y)^2}\\ &= \frac{4y}{(3x+2y)^2}\,.\end{aligned}$$

2) Use implicit differentiation to find $\frac{\partial z}{\partial x}$ if z is defined by the equation $x^2y+xy^2+z+z^3xy=1$.

Solution: $\dfrac{\partial z}{\partial x} = -\dfrac{2xy+y^2+z^3y}{1+3z^2xy}$.

We differentiate the equation $x^2y+xy^2+z+z^3xy=1$ with respect to x and get

$$2xy+y^2+\frac{\partial z}{\partial x}+z^3y+3z^2xy\frac{\partial z}{\partial x} = 0\,.$$

Next we collect the terms involving $\frac{\partial z}{\partial x}$ on the one side

$$(1+3z^2xy)\frac{\partial z}{\partial x}+3z^2xy\frac{\partial z}{\partial x} = -(2xy+y^2+z^3y)\,.$$

Then we solve for $\frac{\partial z}{\partial x}$ and get

$$\frac{\partial z}{\partial x} = -\frac{2xy+y^2+z^3y}{1+3z^2xy}\,.$$

3) Find the equation of the tangent plane to the surface $z = x^2y+2xy-y^2$ at the point $P(1,1,2)$.

Solution: The equation is $z = 4x+y-3$'

The equation of the tangent plane of a surface $z=f(x,y)$ at the point $P(a,b,c)$ is given by

$$z-c = f_x(a,b)(x-a)+f_y(a,b)(y-b)\,.$$

If possible, one should then try to simplify the solution, as we will see in a moment.

So what we have to do, is to find $f_x(1,1)$ and $f_y(1,1)$ for $f(x,y) = x^2y + 2xy - y^2$. $f_x(x,y) = 2xy + 2y$. Incerting $x = 1$ and $y = 1$ we get $f_x(1,1) = 4$. Next we find that $f_y(x,y) = x^2 + 2x - 2y$. Incerting $x = y = 1$ we get $f_y(1,1) = 1$. Hence, the equation is:

$$z - 2 = 4(x-1) + (y-1) = 4x + y - 5\,.$$

Simplifying gives then:

$$z = 4x + y - 3\,.$$