Math 2057-5 Quiz #2 (Fall 2005) **Name:** Solutions

1) Let $z = x^2 + xy + y^2$, $x = s + t$ and $y = st$.

a) Use the chain rule to find $\partial z/\partial s = \underline{(2x + y) + (x + 2y)t = (2 + t)x + (1 + 2t)y}$

Solution: Note that z is a function of the independent variables s and t:

$$z(x, y) = z(x(s, t), y(s, t)) \,.$$

The chain rule tells us that

$$\frac{\partial z}{\partial s} = \frac{\partial z}{\partial x} \cdot \frac{\partial x}{\partial s} + \frac{\partial z}{\partial y} \cdot \frac{\partial y}{\partial s} \,.$$

$$\frac{\partial z}{\partial x} = 2x + y \quad \frac{\partial z}{\partial y} = x + 2y \quad \frac{\partial x}{\partial s} = 1 \quad \frac{\partial y}{\partial t} = t \,.$$

Putting all of this together we get

$$\frac{\partial z}{\partial s} = (2x + y) + (x + 2y)t = (2 + t)x + (1 + 2t)y \,.$$

b) Evaluate $\partial z/\partial s$ at $s = -1$ and $t = 1$. $\partial z/\partial s|_{s=-1,t=1} = \underline{-3}$

Solution: We use the solution to (a). If $s = -1$ and $t = 1$, then $x = -1 + 1 = 0$ and $y = (-1) \cdot 1 = -1$. Hence

$$\partial z/\partial s|_{s=-1,t=1} = (2 \cdot 0 - 1) + (0 + 2(-1))1 = -1 - 2 = -3 \,.$$

2) Find $\partial y/\partial x$ if $y^5 + x^2y^3 = 1 + ye^x$. $\partial y/\partial x = \underline{-\frac{2xy^3 - ye^x}{5y^4 + 3x^2y^2 - e^x} = \frac{ye^x - 2xy^3}{5y^4 + 3x^2y^2 - e^x}}$.

Solution: Recall the following rule: If y is defined implicitly by $F(x, y) = 0$ and $F_y(x, y) \neq 0$, then $dy/dx = -F_x/F_y$. Before we use this formula, we have to bring the equation defining y into the correct form:

$$F(x, y) = y^5 + x^2y^3 - 1 - ye^x = 0$$

We then calculate:

$$F_x = 2xy^3 - ye^x$$

and

$$F_y = 5y^4 + 3x^2y^2 - e^x \,.$$

Hence

$$\frac{dy}{dx} = -\frac{2xy^3 - ye^x}{5y^4 + 3x^2y^2 - e^x} = \frac{ye^x - 2xy^3}{5y^4 + 3x^2y^2 - e^x} \,.$$