Math 2025, Quiz #4 **Name:** ______________________________

1) Determine which of the following sets is **not** a vector space. Explain your answer:

a) $\{(x,y,z) \in \mathbb{R}^3 \mid x+y-z=0\}$.

b) $\{(x_1,x_2,x_3,x_4) \in \mathbb{R}^{10} \mid x_1+x_2-x_3+5x_4=4\}$;

c) $\{f \in C^\infty(\mathbb{R}) \mid f'+f=0\}$;

d) $\{x \in V \mid T(x)=y\}$ where V and W are vector spaces, $T: V \to W$ is linear and $y \in W$.

2) Which of the following maps is **not** linear. Explain your answer:

a) $T: \mathbb{R}^3 \to \mathbb{R}^2$, $T(x,y,z)=(x+2y, x-3y)$;

b) $T: \mathbb{R}^3 \to \mathbb{R}$, $T(x,y,z)=2x+3y+z+1$;

c) $T: C([a,b]) \to \mathbb{R}$, $T(f)=\int_a^b f(t)\ dt$

d) $T: C^\infty(\mathbb{R}) \to C^\infty(\mathbb{R})$, $T(f)=f' \cdot f$.

3) Evaluate the given linear map at the given vector

a) $T: C^\infty(\mathbb{R}) \to C^\infty(\mathbb{R})$, $T(f)=2f'+3f$, $f(x)=x^2$;

b) $T: \mathbb{R}^2 \to \mathbb{R}^3$, $T\left(\begin{pmatrix} x \\ y \end{pmatrix}\right) = \begin{pmatrix} 2 & 1 \\ -1 & 2 \\ 2 & -2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$, $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$;

c) $T: C([0.1]) \to \mathbb{R}$, $T(f)=\int_0^1 f(x)dx$, $f(x)=\cos(2\pi x)$.

d) $T: \mathbb{C}^3 \to \mathbb{C}$, $T(z_1,z_2,z_3)=z_1+2z_2+(3+i)z_3$, $(z_1,z_2,z_3)=(1+i, 1-i, 2+3i)$.