1[3P]) Find the gradient vector field corresponding to the potential function \( \varphi(x) = \frac{1}{\sqrt{x^2 + y^2 + z^2}} \).

**Solution:** The gradient is

\[
\nabla \varphi(x) = \left< \frac{\partial \varphi}{\partial x}(x), \frac{\partial \varphi}{\partial y}(x), \frac{\partial \varphi}{\partial z}(x) \right>.
\]

We have

\[
\frac{\partial}{\partial x} \left( \frac{1}{\sqrt{x^2 + y^2 + z^2}} \right) = \frac{\partial}{\partial x} (x^2 + y^2 + z^2)^{-1/2} = -\frac{1}{2} \cdot 2x \cdot (x^2 + y^2 + z^2)^{-3/2} = \frac{x}{(x^2 + y^2 + z^2)^{3/2}}.
\]

Then doing the same calculation for the other variables shows that

\[
\nabla \varphi(x) = \left< -\frac{x}{(x^2 + y^2 + z^2)^{3/2}}, -\frac{y}{(x^2 + y^2 + z^2)^{3/2}}, -\frac{z}{(x^2 + y^2 + z^2)^{3/2}} \right> = -\frac{x}{|x|^3}.
\]

2[3.5P]) Evaluate the line integral \( \int_C xy \, ds \) where \( C \) is the arc of the parabola \( y = x^2 \) from \((-1, 1)\) to \((1, 1)\).

**Solution:** We use \( x \) as parameter, so the curve is described by \( C: x = x, y = x^2, -1 \leq x \leq 1 \), and \( ds = \sqrt{1 + 4x^2} \, dx \). The integral is therefore

\[
\int_C xy \, ds = \int_{-1}^{1} x^3 \sqrt{1 + 4x^2} \, dx.
\]

The function \( x \mapsto x^3 \sqrt{1 + 4x^2} \) is odd and we integrate from \(-1\) to \(1\). Hence the integral is zero. The final answer is therefore

\[
\int_C xy \, ds = 0.
\]

3[3.5P]) Evaluate the line integral \( \int_C xy \, dx + (x - y) \, dy \) where \( C \) consists of the line segment from \((0, 0)\) to \((1, 2)\).

**Solution:** The line segment can be parametrized as

\[
x = t, \ y = 2t, \ 0 \leq t \leq 1.
\]
Then $dx = dt$ and $dy = 2dt$. The integral is therefore

$$
\int_C xy\,dx + (x - y)\,dy = \int_0^1 (t \cdot 2t + 2(t - 2t))\,dt
$$

$$
= \int_0^1 2t^2 - 2t\,dt
$$

$$
= \left[ \frac{2t^3}{3} - t^2 \right]_0^1
$$

$$
= \frac{2}{3} - 1
$$

$$
= -1/3.
$$

The final answer is therefore

$$
\int_C xy\,dx + (x - y)\,dy = -1/3.
$$