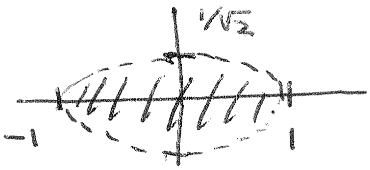
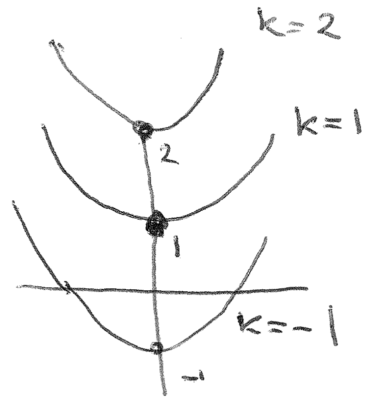
Math 2057-5, Test # 1. Fall 2005 Name: Solutions

1[4P]) Find the domain of the function $f(x, y) = \ln(1 - x^2 - 2y^2)$. $D = \{(x,y) \mid x^2 + 2y^2 < 1\}$

ln(t) is only defined for $t > 0$, so we must have $1 - x^2 - 2y^2 > 0$ or $1 > x^2 + 2y^2$. The picture is





2[6P]) Draw the level curves for the function $f(x, y) = y - x^2$ corresponding to the values 1, 2 and -1. Mark clearly which curve correspond to what value.

$f(x,y) = y - x^2 = k$, so $y = x^2 + k$ which is a parapola

3[24P]) Find the limit, if it exists, or show that the limit does not exists:

a) $\lim_{(x,y)\to(1,1)} \frac{x^2 + 2xy - 1}{x^2 - y^2} =$ DNE

inserting $x=1$, $y=1$, we get $x^2 + 2xy - 1 \to 1 + 2 - 1 = 2$
$x^2 - y^2 \to 1 - 1 = 0$

which results in $\frac{2}{0}$ which does not exists as a limit

b) $\lim_{(x,y)\to(2,1)} \frac{x + 2xy^2 - 3xy}{xy - x + y^2} = \frac{0}{1} = 0$

inserting $x = 2, y = 1$ we get
$x + 2xy^2 - 3xy \to 2 + 4 - 6 = 0$
$xy - x + y^2 \to 2 - 2 + 1 = 1$

c) $\lim_{(x,y)\to(0,0)} \frac{xy}{\sqrt{x^2 + y^2}} = 0$

$x = r\cos\theta$
$y = r\sin\theta$

$$\lim_{(x,y)\to(0,0)} \frac{xy}{\sqrt{x^2+y^2}} = \lim_{r\to 0} \frac{r^2\cos\theta\sin\theta}{r}$$

$$= \lim_{r\to 0} r\cos\theta\sin\theta = 0$$

4[8P]) Where is the function $f(x,y) = \begin{cases} \frac{x^2-y^2}{x^2+y^2} & : \quad (x,y) \neq (0,0) \\ 0 & : \quad (x,y) = (0,0) \end{cases}$ continuous. (Correct argument counts for 4 points.) f is continuous except at (0,0)

If $(x,y) \neq (0,0)$ then $f(x,y)$ is a rational function, so f is continuous at $(x,y) \neq (0,0)$ [because $x^2+y^2 = 0$ only for $x=y=0$]

~~Taking~~ To see if $\lim_{(x,y)\to(0,0)} f(x,y)$ exists we use polar coordinates

$$\frac{r^2(\cos^2\theta - \sin^2\theta)}{r^2} = \cos^2\theta - \sin^2\theta$$ which depends on θ. Hence the limit does not exists, so f is not continuous at (0,0).

5[14P]) Find the given partial derivative of the following functions:

a) $f(x,y) = 2xe^{xy^2}$. $f_y(x,y) = 4x^2e^{xy^2}$

b) $f(x,y) = \frac{x}{x^2+2xy}$. $f_x(x,y) = -\frac{x^2}{(x^2+2xy)^2} = -\frac{1}{(x+2y)^2}$.

solution 1 $f_x(x,y) = \frac{(x^2+2xy) - x(2x+2y)}{(x^2+2xy)^2} = -\frac{x^2}{(x^2+2xy)^2}$.

Solution 2: Simplify: $f(x,y) = \frac{1}{x+2y}$, so

$f_x(x,y) = \frac{-1}{(x+2y)^2}$

6[8P]) Let $f(x,y) = y\ln(2x+y)$. Find the partial derivative $f_{xy}(x,y) = \frac{4x}{(2x+4)^2}$

$f_x(x,y) = \frac{2y}{2x+y}$

$f_{xy}(x,y) = \frac{2(2x+y) - 2y(1)}{(2x+y)^2} = \frac{4x}{(2x+y)^2}$

7[16P]) Let $z = \sqrt{x}e^y$.

a) Find the equation of the tangent plane at the point $(1, 0, 1)$. $\boxed{z - 1 = \frac{1}{2}(x-1) + y}$

$\frac{\partial z}{\partial x} = \frac{e^y}{2\sqrt{x}}$; $x = 1, y = 0$ $\frac{\partial z}{\partial x}(1,0) = \frac{1}{2}$ or

$\frac{\partial z}{\partial y} = \sqrt{x}\, e^y$ $\frac{\partial z}{\partial y}(1,0) = 1$ $\boxed{z = \frac{1}{2}x + y + \frac{1}{2}}$

b) Use the linear approximation to evaluate $\sqrt{0.8}e^{0.2} \simeq$ 1.1

Insert $x = 0.8$ and $y = 0.2$ into the linear approximation $z = \frac{1}{2}x + y + \frac{1}{2}$ to get $z \approx \frac{1}{2}0.8 + 0.2 + 0.5$

$= 0.4 + 0.2 + 0.5 = 1.1$

8) Let $z = x^2y + xy^2$, $x = s + t$ and $y = st$.

a [8P]) Find $\frac{\partial z}{\partial s} =$ $(2xy + y^2) + (x^2 + 2xy)t$

$\frac{\partial z}{\partial x} = 2xy + y^2$ $\frac{\partial x}{\partial s} = 1$. The chain rule gives

$\frac{\partial z}{\partial y} = x^2 + 2xy$ $\frac{\partial y}{\partial s} = t$. $\frac{\partial z}{\partial s} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial s}$

b[4P]) Evaluate $\boxed{\frac{\partial z}{\partial s} = 1}$ at $s = 1$ and $t = -1$.

If $s = 1$ and $t = -1$, then $x = 0$ and $y = -1$. Insert into the formula from (a) gives $\frac{\partial z}{\partial s} = 1$.

9[8P]) Find $\partial z/\partial y$ if z is defined implicitly by $x + y^3 + z^2 = 1 - 6xz$. $\frac{\partial z}{\partial y} = -\frac{3y^2}{2z + 6x}$

If $F(x, y, z) = 0$ and $F_z \neq 0$, then

$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z}$. We have $F(x, y, z) = x + y^3 + z^2 + 6xz - 1 = 0$

$F_y = 3y^2$

$F_z = 2z + 6x$