1[24P]) Let \( c(t) = (t^2 + 1, t^2 - 2t) \).

a) Find the \( x \) and \( y \) coordinates at time \( t = 2 \). \( x = \frac{5}{2}, y = 0 \)

b) Evaluate \( \frac{dy}{dx} \) at the point \( t = 2 \). \( \frac{dy}{dx} = \frac{1}{2} \).

\[
\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2t - 2}{2t}, \quad t = 2 \quad \Rightarrow \quad \frac{2}{4} = \frac{1}{2}
\]

c) The equation of the tangent line through the point \( c(2) \) is \( y - y_0 = m(x - x_0) \) :

\[
y = \frac{1}{2}(x - 5) = \frac{1}{2}x - \frac{5}{2}
\]

2[7P]) The curve \( r = \frac{10}{2 \cos(\theta) - \sin(\theta)} \) represents a line. The equation of the line in form of \( x \) and \( y \) is:

\[
10y = 2x + 10
\]

```
multiply 2 \cos(\theta) \sin(\theta) = 10, 2x - y = 10
```

3[27P]) A circle \( C \) has center at the origin and radius 2. The circle \( K \) has radius 2 and center at the point \((0, 2)\).

a) Write the equation of both circles in polar coordinates.

\( C \) has the equation \( r = 2 \) and \( K \) has the equation \( r = 4 \sin(\theta) \).

b) Find the \( x, y \) coordinates of the points where the two circles intersect.

\[
x = \pm \sqrt{3}, \quad y = 1
\]

\[
2 = 4 \sin(\theta), \quad \sin(\theta) = \frac{1}{2}, \quad \cos(\theta) = \frac{\sqrt{3}}{2}, \quad x = 2 \cos \theta, \quad y = 2 \sin(\theta)
\]

c) Set up the integral representing the area outside the circle \( C \) and inside the circle \( K \).

\[
\theta = \sin^{-1}(1/2) = \frac{\pi}{6}
\]

\[
\text{Area} = \int_{\pi/6}^{3\pi/2} 16 \sin^2(\theta) - 4 \, d\theta
\]
4) [9P]) Set up the integral for the length of the curve $r = 1 + \cos(\theta), 0 \leq \theta \leq 2\pi$.

$$ \sqrt{2} \int_{\theta_0}^{\theta_1} \sqrt{1 + \cos^2 \theta} \, d\theta $$

Length = $\int_{\theta_0}^{\theta_1} \sqrt{(r')^2 + r^2} \, d\theta$, $r'_1 = -\cos(\theta)$

$$ r^2 = 1 + \cos^2 \theta + \sin^2 \theta $$

5) [10P]) The equation of the ellipse that has a center at $(6, 3)$, a focus at $(2, 3)$, and a vertex at $(1, 3)$ is:

$$ \frac{(x-6)^2}{25} + \frac{(y-3)^2}{9} = 1 $$

$c = 6 - 2 = 4$

$a = 6 - 1 = 5$

$b = a^2 - c^2 = 25 - 16 = 9$

6) [18P]) Suppose that $P = (2, 1, 1)$ and $Q = (-1, 3, 1)$.

a) Find $PQ = \langle -3, 2, 0 \rangle$ and $\|PQ\| = \sqrt{13}$

$$ q + 4 = 13 $$

b) The equation of the line through the points $P$ and $Q$ is $\langle 2, 1, 1 \rangle + t \langle -3, 2, 0 \rangle$ or

$$ x = 2 - 3t$$
$$ y = 1 + 2t$$
$$ z = t $$

7) [18P]) Given two lines with parameters $r_1(t) = (10, 6, 5) + t(-2, -2, 1)$ and $r_2(t) = (3, 5, 1) + t(1, -1, 2)$.

a) Find the point of intersection, $P$, of the lines $r_1$ and $r_2$. $P = (6, 2, 7)$

$$ r_1(t) = \langle 10 + 2t, 6, 5 + t \rangle $$

$$ r_2(t) = \langle 3 - 2t, 5 - t, 1 + 2t \rangle $$

Solve $r_1 = r_2$

$$ 2 = 2 - 4t $$
$$ 4 = 7 - 4t $$

b) Write the equation of the line through the point $Q = (1, 1, 1)$ and parallel to the line given by $r_2(t)$.

$$ \vec{r}_3(t) = \langle 1, 1, 1 \rangle + t \langle -2, -2, 1 \rangle $$

Note: $\vec{r}_i$ was changed to $\vec{r}_i(t) = \langle 6, 6, 2 \rangle + t\langle -2, -2, 1 \rangle$. Now the solution is $t = s = 1$ so $P = (4, 4, 1, 3)$
8[18P]) Let \( \mathbf{u} = (2, 1, 1) \) and \( \mathbf{v} = (1, -1, 1) \).

a) The unit vector \( \mathbf{e}_u \) in the direction of \( \mathbf{u} \) is 
\[
\mathbf{e}_u = \frac{\mathbf{u}}{||\mathbf{u}||} \quad , \quad ||\mathbf{u}|| = \sqrt{4+1+1} = \sqrt{6}
\]

b) Find the projection \( \text{proj}_u(\mathbf{v}) = \langle \frac{2}{3}, \frac{1}{3}, \frac{1}{3} \rangle \)
\[
\text{proj}_u(\mathbf{v}) = (\mathbf{v} \cdot \mathbf{e}_u) \mathbf{e}_u = \frac{1}{6} (2, -1, 1) \cdot (2, 1, 1)
\]

9[9P]) Write the equation of the line through the point \( P = (1, -1, 2) \) with normal \( \mathbf{n} = (1, -2, 1) \).

The equation is \( \frac{Z - x}{2} = \frac{y - (-1)}{3} = \frac{z - 2}{5} = 10 \).

Equation: \( \mathbf{n} \cdot (\mathbf{x} - \mathbf{x}_0) = 0 \) \( \text{or} \) \( \mathbf{n} \cdot \mathbf{x} = \mathbf{n} \cdot \mathbf{x}_0 = 1 + 2 + 2 = 5 \)
\[
x - 2y + z = 1 + 2 + 2 = 5
\]

10[10P]) Find the equation of the plane through the points \( P = (1, 0, 2), Q = (1, 1, -2) \) and \( R = (-1, 1, 0) \).

\[
\mathbf{n} = \overrightarrow{PQ} \times \overrightarrow{PR} = \langle 0, 1, 0 \rangle \times \langle -2, 1, -2 \rangle \text{ or any other vector orthogonal to } \langle 0, 1, 0 \rangle, \langle -2, 1, -2 \rangle.
\]

Any such vector is \( \langle 1, 0, -1 \rangle \). So the equation is
\[
x - z = -1 \text{ or } -x + z = 1
\]