Very Preliminary Program

Two of the participants will give Colloquium talks before the meeting. The workshop it self starts on Friday morning. All talks will take place in **Lockett, 277**. The room is equipped with multi-media. Note, *this is a workshop, and there is more weight on questions, discussion and comments, than follow exactly the time schedule!* Talks might therefore be moved. There is a long break on Saturday for those who want to see the **Spanish Town Parade**, which starts downtown at Noon on February, 25$^{\text{th}}$, see *http://www.spanishtownparade.com.*

Wednesday		
Time	Speaker	Title
3:40-4:30	Dorin Dutkay	Colloquium talk: Wavelets and self-similarity
Thursday		
Time	Speaker	Title
3:40-4:30	David Larson	Colloquium talk: Wavelet Sets and Interpolation

Friday		
Time	Speaker	Title
9:40-10:30	Peter Massopust and Dave Larson	Wavelet sets, fractal surfaces, and Coxeter groups, I
10:40-11:30	Peter Massopust and Dave Larson	Wavelet sets, fractal surfaces, and Coxeter groups, II
Break until 2:40		
1:40-2:30	Darrin Speegle	Riesz sequences of exponentials
2:40-3:30	Eugen J. Ionascu	Wavelet sets associated to dilations generated by non-expansive matrices
Break		
4:00 - 5:00	Myung-Sin Song	Mathematical Insights of Wavelet Image Compression

Saturday		
Time	Speaker	Title
9:00-9:50	Palle Jorgensen and Dorin Dutkay	Computational features of wavelet algorithms adapted to a general class of problems from dynamics, I
Break		
10:10-11:00	Palle Jorgensen and Dorin Dutkay	Computational features of wavelet algorithms adapted to a general class of problems from dynamics, II
Break until 2:40		
2:40-3:30	Bradley Currey	Fourier transform and admissibility for non-commutative domains
3:40-4:10	Jasson Vindas	
Break		
4:40-5:10	David Jimenez	PCM Quantization Errors and the White Noise Hypothesis
5:20-5:40	Jens Christensen	Smooth representations and modulation spaces
7-∞	Dinner	

Sunday		
9:30-9:50	Juan Romero	Non-separable Frame Multiresolution Analysis and Fast Wavelet Algorithms in Multidimensions
10:00-10:20	Mihaela Dobrescu	Multiresolution Analysis and Rotation Groups
11:00-11:20	Palle Jorgensen and Dorin Dutkay	Final remarks, open problems and discussion on “Computational features of wavelet algorithms adapted to a general class of problems from dynamics”.
11:30-12:00	David Larson and Peter Massopust	Final remarks, open problems and discussion on “Wavelet sets, fractal surfaces, and Coxeter groups”

Confirmed Participants

Simon Alexander, University of Houston; simon@math.uh.edu (Gradstudent)
Mihaela Dobrescu; Furman University; dobrescu@math.lsu.edu
Jens Christensen, LSU; vepjan@math.lsu.edu (Gradstudent)
Bradley Currey, SLU; curreybn@slu.edu
Dorin Dutkay, Rutgers; ddutkay@math.rutgers.edu
Eugen J. Ionascu, Georgia; ionascu@math.gatech.edu
David Jimenez, Georgia Tech; djimenez@math.gatech.edu (Gradstudent)
Palle Jorgensen, Iowa; jorgen@math.uiowa.edu
David Larson; Texas A&M; larson@math.tamu.edu
Peter Massopust,
Gestur Ólafsson, LSU; olafsson@math.lsu.edu
Juan Romero, University of Houston; jrr@math.uh.edu (Gradstudent)
Myung-Sin Song, Southern Illinois University Edwardsville, msong@siue.edu
Darrin Speegled, SLU; speegled@slu.edu
Jasson Vindas, LSU, jvindas@math.lsu.edu (Gradstudent)

Speakers, Titles and abstracts

Jens Christensen

Smooth representations and modulation spaces

Brad Currey

Fourier transform and admissibility for non-commutative domains

Abstract: Let N be a connected, simply connected nilpotent Lie group with Lie algebra $\mathfrak{n}$, and let H be a closed subgroup of $\mathrm{Aut}(N)$, regarded also in a natural way as a closed subgroup of $\mathrm{GL}(\mathfrak{n}^*)$. Given f an L^2-function on N, define $(\tau(x)f)(y) = f(x^{-1}y),\ x \in N$ and $\tau(h)f(y) = f(h^{-1}y)\det(h)^{-1/2},\ h \in H$. Define $W_\psi(f) = \langle f, \tau(\cdot)\psi\rangle,\ f, \psi \in L^2(N)$. One says that H is admissible if there is $\psi \in L^2(N)$ such that

$$W_\psi : L^2(N) \to L^2(N \rtimes H)$$

is an isometry. When investigating the question of whether H permits admissible vectors, many of the familiar calculations in the case $N = \mathbb{R}^n$ go through if one can construct a submanifold Λ of $\mathfrak{n}^*$ which serves as a domain for the non-commutative Fourier transform, and for which $H(\Lambda) \subset \Lambda$.
We describe a new domain Λ for the general nilpotent Fourier transform for which $H(\Lambda) \subset \Lambda$ holds for a reasonably large class of H. We illustrate by example its use in the construction of admissible vectors.

Mihaela Dobrescu

Multiresolution Analysis and Rotation Groups

Abstract: We present a method of constructing scaling sets and the associated multiresolution analysis wavelets corresponding to rotation groups as dilations.

Speaker: D. Dutkay

Colloquium
Wavelets and self-similarity

Abstract: In the past twenty years the theory of wavelets has proved to be extremely successful, with important applications to image compression and signal processing. The theory involves the construction of orthonormal bases in euclidian spaces generated by translations and dilations. A key feature of these constructions is the property of self-similarity. We exploit this property and, using operator algebra methods, we offer a wider perspective on the subject. We show how techniques from the theory of wavelets can be used in many other contexts such as fractals, dynamical systems, or endomorphisms of von Neumann algebras. Thus, we can construct rich multiresolution structures with scaling functions and wavelets on fractals, solenoids, super-wavelets for Hilbert spaces containing $L^2(\mathbb{R})$, or harmonic bases on fractal measures.

Dorin Dutkay and Palle Jorgensen

A tutorial for general audience.

Computational features of wavelet algorithms adapted to a general class of problems from dynamics.

Abstract: In the talks we begin by recalling a list of reasons why wavelet tools are successful in the familiar setting of square-integrable functions in one or more real variables, i.e., for the analysis of functions in $L^2(\mathbb{R})$ or in $L^2(\mathbb{R}^d)$, $d > 1$. Signal and image processing connections will be mentioned. In the process we recall both old and traditional wavelet bases, as well as some newer and non-traditional wavelet decompositions. We also recall the notion of a multiresolution system in the traditional case of function theory. We then proceed to take a wider and more geometric view of the standard definition of multiresolution system for $L^2(\mathbb{R})$ or even $L^2(\mathbb{R}^d)$, $d > 1$. As is well known, in the standard definition of multiresolution system, the scaling operation is typically fixed by a decision to scale functions by powers of 2, by powers of an integer (one variable); or if $d > 1$, by powers of a fixed integral and expanding matrix. This choice makes it natural to use translations by the integers, or by integral lattices in $\mathbb{R}^d$. However, we shall show that there are much more general settings from dynamics, e.g., affine iterated function systems IFSs, standard Cantor fractals, Julia sets and Julia dynamics, and fractional Brownian motion, where essential ideas and algorithms from traditional wavelet theory in fact carry over, mutatis mutandis. Indeed, relying on some amount of operator theory, in this context we get Hilbert spaces and usable multiresolution systems. But there are some important differences between the old and the new which will be outlined, and discussed. One thing that does carry over to our new setting of these general dynamical systems is that we get wavelet bases; and

that we are able to compute wavelet expansions based on the same kind of iterative matrix algorithm which has served us so well for the more standard wavelet settings.

Eugen J. Ionascu

Wavelet sets associated to dilations generated by non-expansive matrices

David Jimenez

PCM Quantization Errors and the White Noise Hypothesis

Abstract: The White Noise Hypothesis (WNH), introduced by Bennett half century ago, assumes that in the pulse code modulation (PCM) quantization scheme the errors in individual channels behave like white noise, i.e. they are independent and identically distributed random variables. The WNH is key to estimating the means square quantization error (MSE). But is the WNH valid? In this paper we take a close look at the WNH. We show that in a redundant system the errors from individual channels can never be independent. Thus to an extend the WNH is invalid. Our numerical experients also indicate that with coarse quantization the WNH is far from being valid. However, as the main result of this paper we show that with fine quantizations the WNH is essentially valid, in which the errors from individual channels become asymptotically *pairwise* independent, each uniformly distributed in $[-\Delta/2, \Delta/2)$, where Δ denotes the stepsize of the quantization.

Dave Larson

Colloquium
Wavelet Sets and Interpolation

Abstract: A wavelet is a special case of a vector in a separable Hilbert space that generates a basis under the action of a collection, or "system", of unitary operators defined in terms of translation and dilation operations. We will begin by describing an operator-interpolation approach to wavelet theory using the local commutant of a unitary system that was developed by the speaker and his collaborators a few years ago. This is really an abstract application of the theory of operator algebras, mainly von Neumann algebras, to wavelet theory. The concrete applications of operator-interpolation to wavelet theory include results obtained using specially constructed families of wavelet sets. We will discuss some unpublished and partially published results, and some brand new results, that are due to this speaker and his former and current students, and other collaborators.

David Larson and Peter Massopust

Tutorial for general audience.

Wavelet sets, fractal surfaces, and Coxeter groups

Abstract: We loosely define a wavelet set as a measurable subset of $\mathbb{R}^n$ that generates a partition of $\mathbb{R}^n$ under the action of a generalized dilation group, i.e. it is the fundamental domain of this group, and which supports a natural orthonormal basis (or Riesz basis)

structure. This is the case with the usual dilation-translation wavelet sets. In the cases we are concerned with in this tutorial the theory is even richer: a wavelet set will in fact support an entire, natural, multiresolution analysis-type structure. In the setting where the fundamental domain is a foldable figure, the underlying group is a Coxeter group and fractal surfaces defined on the foldable figure provide a natural multiresolution analysis structure on $L^2(\mathbb{R}^n)$.
We briefly discuss wavelet sets and present some of their properties and introduce the construction of fractal surfaces using a specific class of iterated function systems. Definitions and results from the theory of Coxeter groups and affine Weyl groups are also given.

Juan Romero

Non-separable Frame Multiresolution Analysis and Fast Wavelet Algorithms in Multidimensions.

Abstract: We extend some of the classical results of the theory of frame multiresolution analysis in multidimensions. We focus on non-separable structures. We use these frame MRAs to derive Fast Wavelet algorithms. At the end, we will show how this matrix equation leads to a generalization of the Unitary Extension Principle.

Myung-Sin Song

Mathematical Insights of Wavelet Image Compression

Abstract: Wavelets provide a powerful and remarkably flexible set of tools for handling fundamental problems in science and engineering, such as signal compression, fingerprint compression, image de-noising, image enhancement, image recognition, and image compression to name a few. In my talk, I am going to concentrate on wavelet application in the field of Image Compression in order to observe how wavelet is implemented to be applied to an image in the process of wavelet decomposition process to enhance the compression, and also how mathematical aspects of wavelet affect the wavelet decomposition process to produce in different compression results. Wavelet image compression is performed with various known wavelets with different mathematical properties such as vanishing moments, symmetry, orthogonality and biorthogonality. I will talk about the insights of how wavelets in mathematics are implemented in a way to fit the engineering model of image compression.

Darrin Speegle

Riesz sequences of exponentials

Abstract: Let $E \subset \mathbb{T}$. It is an open question whether $\{e^{2\pi ikx}1_E : k \in \mathbb{Z}\}$ can be partitioned into a finite number of Riesz sequences. It turns out that this question acts as a good organizing problem for understanding several important results in harmonic analysis. In this talk, I will present both the important results and their applications to the problem mentioned above.
To be announced.