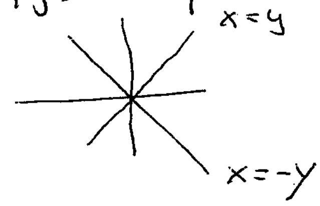
Math 2058-1, Test # 1. Fall 2005 **Name:** Solutions

1[3P]) Find the domain of the function $f(x, y) = x^2 \ln(x - y)$. $x > y$

$D = \{ (x, y) \mid x > y \}$

2[4P]) Find the largest set where the function $f(x, y) = \dfrac{1}{x^2 - y^2}$ is continuous.

f is continuous outside the set $\{ (x, y) \in \mathbb{R}^2 \mid x \neq \pm y \}$



3[18P]) Find the limit, if it exists, or show that the limit does not exists:

a) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2 - y^2}{x^2 + y^2} =$ DNE.

$x = r\cos\theta$
$y = r\sin\theta$

$\cos^2\theta - \sin^2\theta$. Depends on θ

b) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{xy^2}{x^2 + y^2} =$ 0

Use polar coordinates: $\dfrac{r^3 \cos\theta \sin^2\theta}{r^2} = r\cos\theta\sin^2\theta \to 0$

c) $\displaystyle\lim_{(x,y,z)\to(0,0,0)} \frac{xy + yz^2 + xz^2}{x^2 + y^2 + z^2} =$ DNE

Let $z = 0$: $\dfrac{xy}{x^2 + y^2}$ $x = r\cos\theta$, $y = r\sin\theta$ $\dfrac{r^2\cos\theta\sin\theta}{r^2} = \cos\theta\sin\theta$ Depends on θ.

4[14P]) Find the indicated partial derivatives:

a) $f(x, y) = \sin\left(\dfrac{x}{x + y}\right)$. $f_y =$ $\dfrac{-x}{(x+y)^2} \cos\left(\dfrac{x}{x+y}\right)$

b) $f(x, y) = 3x^2y + 2e^{xy}$. $f_{xy}(x, y) =$ $6x + 2e^{xy}(1 + xy)$

$f_x(x,y) = 6xy + 2ye^{xy}$

$f_{xy}(x,y) = 6x + 2e^{xy} + 2xye^{xy}$

5[14P]) Suppose $f(x,y) = (1+xy)^{3/2}$

a) Find the linear approximation of $f(x,y)$ at the point $(4,2)$.

$L = 9(x-4) + 18(y-2) + 27 = 9x + 18y - 45$

$f(4,2) = 27$

$f_x = \frac{3}{2} y (1+xy)^{1/2}; \quad f_x(4,2) = \cancel{3 \cdot 2 \cdot 3} = \frac{3}{2} \cdot 2 \cdot 3 = 9$

$f_y = \frac{3}{2} x (1+xy)^{1/2}, \quad f_y(4,2) = \frac{3}{2} \cdot 4 \cdot 3 = 18$

b) Use the linear approximation to estimate $f(3.99, 2.1) \approx$ 28.7

6) Suppose $z = 4x - y^2$, $x = st$ and $y = e^{st}$.

a[7P]) Find $\frac{\partial z}{\partial t} = 4 \cdot s - 2y s e^{st} = 4s - 2se^{2st}$

$\frac{dx}{dt} = s, \quad \frac{dy}{dt} = se^{st}, \quad \frac{\partial z}{\partial x} = 4, \quad \frac{\partial z}{\partial y} = -2y$

b[4P])) Then compute $\frac{\partial z}{\partial s}(0,2) =$ 0

$\left.\frac{\partial z}{\partial x}\right|_{t=0, s=2} = 4, \quad \left.\frac{\partial z}{\partial y}\right|_{t=0, s=2} = -2, \quad \frac{dx}{ds}(0,2) = 0, \quad \frac{dy}{ds} = 0$

7[24P]) Suppose $f(x,y) = -2x^2 + 3xy$.

a) Find the gradient $\nabla f(x,y) =$ $(-4x + 3y, 3x)$

b) Evaluate the gradient at $(2,1)$, $\nabla f(2,1) =$ $(-5, 6)$

$-4 \cdot 2 + 3 = -5$

$3 \cdot 2 = 6$

c) Find the directional derivative of f at the point $(2,1)$ in the direction of $(5,5)$. $D_{\mathbf{u}}f(2,1) =$ 1.8

$(5,5)-(2,1)=(3,4)$

$\sqrt{9+16} = \sqrt{25}=5$

unit vector $\vec{u}=\frac{1}{5}(3,4)$

directional derivative

$\nabla f(2,1)\cdot\vec{u} = \frac{1}{5}(-5\cdot 3+6\cdot 4) = 1.8$

d) Find the maximum rate of increase of f at $(2,1)$ and the direction in which it occurs.

Maximum rate of change: $\sqrt{61}$ Direction: $\frac{1}{\sqrt{61}}(-5,6)$

$\nabla f = (-5,6)$. Max. change in the direction of the gradient $\frac{1}{\sqrt{25+36}}(-5,6) = \frac{1}{\sqrt{61}}(-5,6)$

8) Find the equation of the tangent plane and normal line at the point $(3,-2,1)$ to the ellipsoid $\underbrace{\frac{x^2}{9}+\frac{y^2}{4}+z^2}_{F(x,y,z)} = 3$.

[6P]) Tangent plane: $2x-y+2z=6$

[5P]) Normal line: $\frac{3}{2}(x-3) = \frac{y+2}{-1} = \frac{z-1}{2}$

$F_x = \frac{2}{9}x$, $x=3$: $\frac{2}{3}$

$F_y = \frac{y}{2}$, $y=-2$: -1

$F_z = 2z$, $z=1$: 2

$0=\frac{2}{3}(x-3)-(y+2)+2(z-1)$

$= 2x-2-y-2+2z-2$

$= 2x-y+2z-6$