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ON THE INTERSECTIONS OF CIRCUITS AND COCIRCUITS IN MATROIDS

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Seymour has shown that a matroid has a triad, that is, a 3-element set which is the intersection of a circuit and a cocircuit, if and only if it is non-binary. In this paper we determine precisely when a matroid M has a quad, a 4-element set which is the intersection of a circuit and a cocircuit. We also show that this will occur if M has a circuit and a cocircuit meeting in more than four elements. In addition, we prove that if a 3-connected matroid has a quad, then every pair of elements is in a quad. The corresponding result for triads was proved by Seymour.

1. Introduction

Several matroid results are concerned with the possible cardinalities of the intersections of circuits and cocircuits. For example, it is well-known that a circuit and a cocircuit in a matroid cannot have exactly one common element, while a matroid is binary if and only if every set which is the intersection of a circuit and a cocircuit has even cardinality. The latter result was sharpened by Seymour [6] who showed that a matroid is binary precisely when it does not have a *triad*, that is, a 3-element set which is the intersection of a circuit and a cocircuit. More recently, Seymour [8] extended this result for 3-connected matroids by proving that if such a matroid M is non-binary, then every pair of elements is contained in a triad and hence is part of the ground set of a $U_{2,4}$ minor of M.

In this paper we investigate further those sets which are the intersection of a circuit and a cocircuit. In particular, we concentrate on such sets having four elements, calling these sets *quads*. In Section 2 we determine when an arbitrary matroid has a quad, showing that this occurs for a 3-connected matroid if and only if both its rank and its corank exceed two. The main result of the section shows that if, for some $k \geqq 5$, the matroid M has a k-element set which is the intersection of a circuit and a cocircuit, then M has a quad. In Section 3 we explicitly determine which matroids have no quads. Finally, in Section 4, we prove that in a 3-connected matroid containing a quad, every pair of elements is in a quad.

The matroid terminology used here will in general follow Welsh [10]. The ground set, rank and corank of the matroid M will be denoted by $E(M)$, rk$M$ and

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cork$M$ respectively. If $T\subseteq E(M)$, then rk$T$ denotes the rank of T. The deletion and contraction of T from M will be denoted by $M\setminus T$ and M/T respectively. Flats of M of ranks one and two will be called *points* and *lines*. The whirl of rank r [10, pp. 80—81] and the uniform matroid of rank k on an n-element set will be denoted by $\mathcal{W}^r$ and $U_{k,n}$.

For $n\geqq 1$, the matroid M is *n-connected* [9, p. 1303] provided that, for all positive integers $k<n$, there is no subset T of $E(M)$ such that $|T|\geqq k$, $|E(M)\setminus T|\geqq k$ and $\text{rk}T+\text{rk}(E(M)\setminus T)-\text{rk}M=k-1$. Thus a matroid is 2-connected precisely when it is connected [10, p. 69]. Moreover,

(1.1) *M is n-connected if and only if M^* is n-connected.* ▮

We shall assume familiarity with the operations of series and parallel connection of matroids, these operations having been discussed in detail in [3]. For matroids M_1 and M_2 such that $E(M_1)\cap E(M_2)=\{p\}$, we shall denote the parallel connection of M_1 and M_2 with respect to the basepoint p as $P((M_1, p), (M_2, p))$ or just $P(M_1, M_2)$. The following fundamental link between 3-connection and parallel connection was proved by Seymour [7, (2.5)].

(1.2) Theorem. *A connected matroid M is not 3-connected if and only if there are matroids M_1 and M_2 each having at least three elements such that $M=P((M_1, p), (M_2, p))\setminus p$ where p is not a loop or a coloop of M_1 or M_2.* ▮

When M decomposes as in this theorem, we call M the *2-sum* of M_1 and M_2. It is routine to check, using the properties of parallel connection, that

(1.3) *The 2-sum of M_1 and M_2 is connected if and only if both M_1 and M_2 are connected.* ▮

If $\{x, y\}$ is a circuit of the matroid M, we say that x and y are *in parallel* in M. If, instead, $\{x, y\}$ is a cocircuit, then x and y are *in series*. A *parallel class* of M is a maximal subset A of $E(M)$ such that if a and b are distinct elements of A, then a and b are in parallel. *Series classes* are defined analogously. A series or parallel class is *non-trivial* if it contains at least two elements. The matroid M' is a *series extension* of M if $M'=M/T$ and every element of T is in series with some element of M not in T. *Parallel extensions* are defined analogously. We call M'' a *series-parallel extension* of M if M'' can be obtained from M by a sequence of operations each of which is either a series or parallel extension. A series-parallel extension of the two-element matroid $U_{1,2}$ is called a *series-parallel network*. A thorough investigation of the properties of such matroids can be found in Brylawski's paper [3].

2. Intersections of circuits and cocircuits

The main result of this section is that if a matroid M has a set with more than four elements which is the intersection of a circuit and a cocircuit, then M has a set with exactly four elements which is the intersection of a circuit and a cocircuit. In addition, an included minor characterization for when a 3-connected matroid has a quad is proved. The first three results are preliminaries generalizing results of Seymour [8, (2.4), (2.5)]. The elementary proof of the first of these is omitted.

(2.1) Lemma. *Let M' be a minor of M and suppose that the set X is the intersection of a circuit and a cocircuit in M'. Then X is the intersection of a circuit and a cocircuit in M.* ▮

(2.2) Proposition. *Let M be a matroid containing a k-element set X which is the intersection of a circuit C and a cocircuit C^*. Then M has a minor N in which X is both a circuit and a cocircuit and such that* $\text{rk}N = k-1 = \text{cork}N$.

Proof. By contracting the elements of $C \setminus C^*$ and deleting the elements of $C^* \setminus C$, we obtain a minor N' of M in which X is both a circuit and a cocircuit. If $E(N') \setminus X$ contains a circuit of N', then we delete an element e from this circuit. The set X remains a circuit in $N' \setminus e$ and, moreover, as $E(N') \setminus (X \cup e)$ is a hyperplane of $N' \setminus e$, the set X is also a cocircuit of $N' \setminus e$. By repeating this operation of deleting single elements from circuits of N' contained in $E(N') \setminus X$, we eventually obtain a matroid N'' in which X is both a circuit and a cocircuit and $E(N'') \setminus X$ is independent.

Now, if $E(N'') \setminus X$ contains a cocircuit of N'', then we contract a single element f from this cocircuit. In N''/f, the set X is still both a circuit and a cocircuit and its complement is still independent. By repeating this operation of contracting single elements from cocircuits of N'' contained in $E(N'') \setminus X$, we eventually obtain the required minor N of M in which X is both a circuit and a cocircuit and $E(N) \setminus X$ is independent in both N and N^*. ▮

Euclidean representations for the 6-element matroids, P_6, Q_6 and R_6, appearing in the next result are given in Figure 1.

(2.3) Corollary. *The set X is a quad in the matroid M if and only if M has a minor isomorphic to one of the matroids $U_{3,6}$, $\mathcal{W}^3$, $M(K_4)$, P_6, Q_6 or R_6 in which X is a quad.*

Proof. If X is a quad in a minor of M, then by Lemma 2.1, X is a quad in M. Now suppose that X is a quad in M. Then, by Proposition 2.2, M has a minor N in which X is both a circuit and a cocircuit and such that $\text{rk}N = \text{cork}N = 3$. It is now routine to check that N must be one of the 6 matroids $U_{3,6}$, $\mathcal{W}^3$, $M(K_4)$, P_6, Q_6 or R_6. ▮

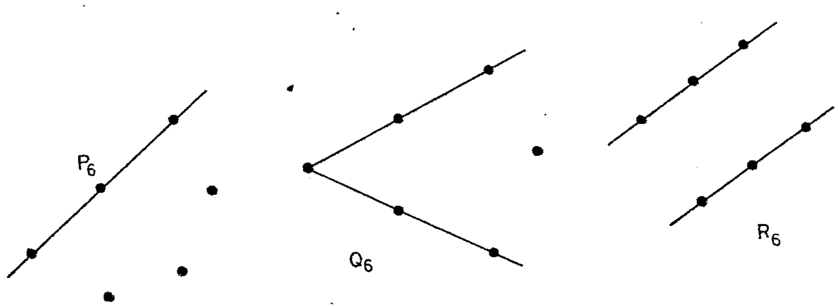

Fig. 1

(2.4) Corollary. *A binary matroid has a quad if and only if it has $M(K_4)$ as a minor.*

Proof. This follows immediately from the preceding corollary upon noting that none of the matroids $\mathcal{W}^3$, $U_{3,6}$, P_6, Q_6 or R_6 is binary. ▮

If a matroid M contains a k-element set X which is the intersection of a circuit and a cocircuit, then clearly X is the intersection of a circuit and a cocircuit in M^*. Thus, for all k, the set $\mathcal{S}_k$ of minor-minimal matroids containing such a k-element set is closed under duality. Moreover, by [8, (2.5)] and Corollary 2.3, when $k=3$ or 4, all the members of $\mathcal{S}_k$ are self-dual. To see that this does not hold in general, consider the rank-four matroid shown in Figure 2. This matroid has $\{1, 2, 4, 6, 7\}$ as both a circuit and a cocircuit and hence is in $\mathcal{S}_5$. However, it is not self-dual because it has two 3-element circuits but only one 3-element cocircuit.

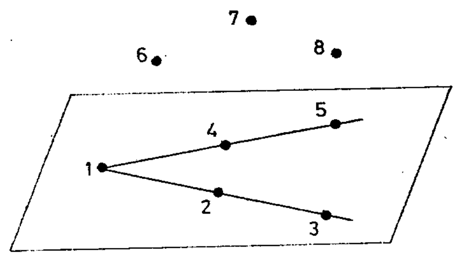

Fig. 2

If M and N are matroids with $M \setminus e=N$, then M is an *extension* of N; if instead, $M/e=N$, then M is a *lift* of N.

(2.5) **Theorem.** *Let M be a 3-connected matroid. Then the following are equivalent.*

(i) *Both the rank and corank of M exceed two.*
(ii) *M has a minor isomorphic to one of $U_{3,6}$, $\mathcal{W}^3$, $M(K_4)$, P_6 or Q_6.*
(iii) *M has a quad.*

Proof. Assume that M has rank and corank exceeding two. Then, by (1.1), M^* is 3-connected, hence by [5, Theorem 4.1] there is a sequence of 3-connected matroids ending with M^* and beginning with the cycle matroid of a wheel, a whirl or $U_{3,5}$ such that each matroid in the sequence is a lift or an extension of its predecessor. Hence there is a similar sequence ending with M and beginning with the cycle matroid of a wheel, a whirl or $U_{2,5}$. In the first two cases, M has $M(K_4)$ or $\mathcal{W}^3$ as a minor. In the third case, M has as a minor a 3-connected lift of a line having at least five points. It is straightforward to check that this lift has one of P_6, Q_6 or $U_{3,6}$ as a minor. We conclude that (i) implies (ii). It is routine to check that each of $U_{3,6}$, $\mathcal{W}^3$, $M(K_4)$, P_6 and Q_6 has a quad. Hence (ii) implies (iii). Finally, since (iii) clearly implies (i), the theorem is proved. ∎

In terms of circuit-cocircuit intersections, Seymour's characterization of non-binary matroids in terms of the existence of a triad [6, p. 360] states that a matroid having a circuit and a cocircuit meeting in an odd number of elements has a circuit and a cocircuit meeting in exactly three elements. The next two results have the same form as this, the first being rather elementary.

(2.6) **Proposition.** *Suppose that M has a k-element set X which is the intersection of a circuit and a cocircuit where $k \geqq 3$. Then, for some t in $\{\lceil k/2 \rceil, \lceil k/2 \rceil+1, \ldots, k-1\}$, M has a t-element set which is the intersection of a circuit and a cocircuit.*

Proof. By Proposition 2.2, M has a minor N having $2k-2$ elements and rank $k-1$ in which X is both a circuit and a cocircuit. Take a $(k-3)$-element subset Y of $E(N)\setminus X$ and consider the hyperplanes $H_1, H_2, \ldots, H_m$ of N containing Y and meeting X. Since X spans N, $m \geqq 2$ and so, for some i, $|H_i \cap X| \leqq \lfloor k/2 \rfloor$. Thus the cocircuit complementary to H_i meets X in a t-element set Z where

$$t \in \{\lceil k/2 \rceil, \lceil k/2 \rceil + 1, \ldots, k-1\}.$$

By Lemma 2.1, Z is the intersection of a circuit and a cocircuit in M. ∎

The next theorem is the main result of this section.

(2.7) Theorem. *Let M be a matroid containing a k-element set X which is the intersection of a circuit and a cocircuit for some $k \geqq 4$. Then M has a quad.*

Proof. We argue by induction on $|E(M)|$. Since $k \geqq 4$, M has rank and corank at least three. Therefore, if M is 3-connected, the result follows by Theorem 2.5. Furthermore, by the induction assumption we may assume that both M and M^* are simple. We now suppose that M is not 3-connected. Then, by Theorem 1.2, $M = P(M_1, M_2)\setminus p$ for some matroids M_1 and M_2, each of which has fewer elements than M. Since X is the intersection of a circuit and a cocircuit of M, there are two possibilities: either one of M_1 and M_2 has a k-set which is the intersection of a circuit and a cocircuit; or, for $i=1, 2$, M_i has a j_i-element set X_i such that $X_i = C_i \cap C_i^*$ where C_i and C_i^* are, respectively, a circuit and a cocircuit of M_i containing p. In the first case, since both M_1 and M_2 are minors of M [7, (2.6)], the result follows on combining the induction assumption with Lemma 2.1. In the second case, $j_1 + j_2 - 2 = k$. Now, as the result holds trivially for $k=4$, we may assume $k \geqq 5$, and so $j_1 + j_2 \geqq 7$. Therefore, one of j_1 and j_2 exceeds 3, and so, by the induction assumption, M_1 or M_2 has a quad. It follows by [7, (2.6)] and Lemma 2.1 that M has a quad. ∎

3. The matroids without quads

It is clear that a disconnected matroid has a quad if and only if one of its components has a quad. Corollary 2.3 gives one characterization of when a matroid has a quad. The next result determines more explicitly precisely which connected matroids have no quads.

(3.1) Theorem. *Let M be a connected matroid having at least three elements. If M has no quads, then M is a series-parallel extension of a uniform matroid of rank or corank two.*

Proof. We argue by induction on $|E(M)|$. If $|E(M)|=3$, then $M \cong U_{1,3}$ or $U_{2,3}$, so the required result holds. Now assume the result true for $|E(M)|<n$ and let $|E(M)| = n$. If M is binary, then, since M has no quads, Corollary 2.4 implies that M has no minor isomorphic to $M(K_4)$. Hence, by [3, Theorem 7.6], M is a series-parallel network. Therefore, as $|E(M)| \geqq 3$, M is a series-parallel extension of $U_{1,3}$ or $U_{2,3}$ and the required result holds.

We can now assume that M is non-binary. Furthermore, we may also suppose that M has no non-trivial series or parallel classes as otherwise the required result follows easily by the induction assumption.

If M is 3-connected, then by Theorem 2.5, $\mathrm{rk}M \leqq 2$ or $\mathrm{cork}M \leqq 2$. But, as M is non-binary, it has $U_{2,4}$ as a minor and therefore has rank and corank at least two. It follows that either $\mathrm{rk}M = 2$ or $\mathrm{cork}M = 2$ and hence M or M^* is uniform of rank two.

Now suppose that M is not 3-connected. Then, by (1.2) and (1.3), $M = P(M_1, M_2)\setminus p$ for some connected minors M_1 and M_2 of M, each of which has at least three elements. As M is non-binary, at least one of M_1 and M_2 is non-binary. Assume that both are non-binary. Then, by [1, Theorem 3.7], M_1 has a $U_{2,4}$ minor using p and M_2 has a $U_{2,4}$ minor using p. It follows that $P(M_1, M_2)\setminus p$ has R_6 as a minor. But R_6 contains a quad and so we obtain the contradiction that M has a quad.

It remains to consider the case when exactly one of M_1 and M_2, say M_1, is non-binary. Then, as M_2 is binary, connected and does not contain a quad, M_2 is a series-parallel network and hence, by [3, Theorem 7.6], is a series-parallel extension of p. It follows that M has a non-trivial series or parallel class and this contradiction completes the proof. ∎

The next result follows immediately on combining the last theorem with Corollary 2.3.

(3.2) Corollary. *Let M be a connected matroid having at least three elements. Then either*

(i) *M has $U_{3,6}$, $M(K_4)$, $\mathcal{W}^3$, P_6, Q_6 or R_6 as a minor;*

or

(ii) *M or M^* is a series-parallel extension of a k-point line for some $k \geqq 3$.* ∎

4. Pairs of elements in quads

An immediate consequence of results of Bixby [1] and Seymour [6] is that if a connected matroid M has a triad, then every element of M is in a triad. If, in addition, M is 3-connected, then Seymour [8, (3.1)] has shown that every pair of elements of M is in a triad. In this section we prove the analogue of Seymour's result for quads and note that the analogue of Bixby's result does not hold. We shall use the following lemma, the straightforward proof of which is omitted.

(4.1) Lemma. *If M is isomorphic to one of the matroids $U_{3,6}$, $\mathcal{W}^3$, $M(K_4)$, P_6 or Q_6, then every pair of elements of M is in a quad.* ∎

(4.2) Theorem. *Let M be a 3-connected matroid having rank and corank at least three. Then every pair of elements of M is in a quad.*

Proof. We shall prove by induction on $|E(M)|$ that every pair of elements of M is in the ground set of some 6-element minor of M which is isomorphic to one of the matroids $U_{3,6}$, $\mathcal{W}^3$, $M(K_4)$, P_6 or Q_6. The theorem then follows immediately from the preceding lemma.

As M is 3-connected having rank and corank at least three, $|E(M)| \geqq 6$ and, by Theorem 2.5, M has a minor isomorphic to one of the 5 specified 6-element matroids. The result therefore follows immediately if $|E(M)| = 6$. Now suppose that $|E(M)| > 6$ and the result is true for all matroids with fewer elements than M. Let $\mathrm{rk}M = 3$. If M is minimally 3-connected, then by [4, Lemma 4.5], either (i) M is isomorphic to a whirl or the cycle matroid of a wheel, or (ii) M/e is minimally 3-connected for some element e of M, or (iii) for some elements f, x and y of M, either $M/f\setminus x$ or $M/f\setminus x, y$ is minimally 3-connected. In the first case, as $\mathrm{rk} M=3$, we obtain the contradiction that $|E(M)|=6$. In cases (ii) and (iii), M/e and M/f both have rank two. But the only minimally 3-connected matroid of rank 2 is $U_{2,3}$ and, since $|E(M)|>6$, none of M/e, $M/f\setminus x$ or $M/f\setminus x, y$ can be isomorphic to $U_{2,3}$. We conclude that M is not minimally 3-connected and therefore has an element z such that $M\setminus z$ is 3-connected. By the induction assumption, for all pairs $\{x_1, x_2\}$ of elements of $E(M\setminus z)$, M has a minor of the required type using both x_1 and x_2. Moreover, as M has rank 3, if $w\in E(M\setminus z)$, M has a restriction using w which is isomorphic to one of the 5 specified matroids. It is now a routine matter to check that by adding z to any of these 5 matroids we must obtain a minor of the required type that uses both z and w. We conclude that the required result holds if M has rank 3 and, by duality, the result also holds if M has corank 3. We shall now assume that M has rank and corank exceeding 3 and show that an arbitrary pair $\{x, y\}$ of elements of M is in the ground set of a minor isomorphic to one of the 5 specified 6-element matroids. If $e\in E(M)\setminus\{x, y\}$ and $M\setminus e$ or M/e is 3-connected, then we can apply the induction assumption to obtain the required result. Thus we may assume, that for all e in $E(M)\setminus\{x, y\}$, neither $M\setminus e$ nor M/e is 3-connected. If, in addition, none of $M\setminus x$, $M\setminus y$, M/x or M/y is 3-connected, then every element of M is essential [9] so, by [9, (8.3)], M is a whirl or the cycle matroid of a wheel. But, in that case, $E(M)$ has a 6-element subset X containing both x and y and M has a minor on X isomorphic to either $M(K_4)$ or $\mathcal{W}^3$. Hence the required result holds. Therefore we may assume that at least one of $M\setminus x$, $M\setminus y$, M/x and M/y is 3-connected.

Now, by [2, Theorem 1], for all elements e of M, either $M\setminus e$ is a series extension of a 3-connected matroid, or M/e is a parallel extension of a 3-connected matroid. We now distinguish four cases:

(i) for some element e_1 of $E(M)\setminus\{x, y\}$, the matroid $M\setminus e_1$ is a series extension of a 3-connected matroid N having rank at least 3, and x and y are in different series classes of $M\setminus e_1$ and hence may be assumed to lie in $E(N)$;

(ii) for all elements e of $E(M)\setminus\{x, y\}$, $M\setminus e$ is a series extension of a 3-connected matroid and x and y are in series in $M\setminus e$;

(iii) for some elements f_1 and f_2 of $E(M)\setminus\{x, y\}$, x and y are in series in $M\setminus f_1$ and are in parallel in M/f_2;

(iv) for some element e_2 of $E(M)\setminus\{x, y\}$, $M\setminus e_2$ is a series extension of a rank-two 3-connected matroid N and $\{x, y\}\subseteq E(N)$.

Evidently one of (i)—(iv) must hold for either M or M^*. In the latter case, we use M^* in place of M in the argument that follows.

In case (i), since $\mathrm{cork} M>3$, $\mathrm{cork} N\geqq 3$ and the required result follows on applying the induction assumption to N.

In case (ii), $\{e, x, y\}$ is a cocircuit of M for all e in $E(M)\setminus\{x, y\}$. Hence M has corank 2; a contradiction.

In case (iii), $\{f_1, x, y\}$ is a cocircuit and $\{f_2, x, y\}$ is a circuit of M. Thus, we get the contradiction that none of $M\setminus x$, $M\setminus y$, M/x or M/y is 3-connected.

In case (iv), since the only rank-two 3-connected matroids are uniform, $N\cong U_{2,k}$ for some $k\geqq 3$. But $\mathrm{cork}(M\setminus e_2)=\mathrm{cork} N=\mathrm{cork} M-1\geqq 3$, so $k\geqq 5$. Now, as $\mathrm{rk} M\geqq 4$ and $\mathrm{rk} N=2$, $M\setminus e_2$ has at least one non-trivial series class. Choose x_1 and x_2 in this class taking x_1 equal to x or y if either x or y is in the class. Contract all the remaining elements from this series class and then contract all but one element from each of the other non-trivial series classes ensuring that the elements x and y are kept during this process. The resulting contraction of M has rank 3 and has $\{e_2, x_1, x_2\}$ as a cocircuit. Now delete all but 3 elements, a, b and c, from the line which is complementary to $\{e_2, x_1, x_2\}$, again ensuring that x and y are kept. The resulting matroid M' has $\{e_2, x_1, x_2\}$ as a cocircuit and $\{a, b, c\}$ as the corresponding hyperplane. Moreover, the closure of $\{x_1, x_2\}$ in M' does not contain a, b or c as $M' \setminus e_2 / x_1$ is isomorphic to a restriction of the simple matroid N; nor does the closure of $\{x_1, x_2\}$ contain e_2, otherwise e_2 is in the closure in M of the series class S_1 of $M \setminus e_2$ containing $\{x_1, x_2\}$. In that case, $\mathrm{rk}(S_1 \cup e_2) = \mathrm{rk} S_1$ and if $S_2 = E(M \setminus e_2) \setminus S_1$, then $\mathrm{rk} M = \mathrm{rk} S_2 + |S_1| - 1$ and $\mathrm{rk} S_1 = |S_1|$. Thus $\mathrm{rk} S_1 + \mathrm{rk} S_2 = \mathrm{rk} M + 1$, so $\mathrm{rk}(S_1 \cup e_2) + \mathrm{rk} S_2 = \mathrm{rk} M + 1$ and this is a contradiction to the fact that M is 3-connected. We conclude that M' is isomorphic to one of P_6 and Q_6 and, as x and y are included among the elements of M', the required result follows. ∎

The next result follows immediately on combining Theorems 2.5 and 4.2.

(4.3) Corollary. *Let M be a 3-connected matroid containing a quad. Then every pair of elements of M is in a quad.* ∎

Seymour [8, p.392] gives an explicit characterization of when two elements in an arbitrary connected matroid M are contained in the ground set of a $U_{2,4}$ minor of M, or equivalently, are contained in a triad of M. The corresponding result holds for when two elements in an arbitrary connected matroid lie in a quad but we shall not give the details.

To see that Corollary 4.3 need not hold if M is not 3-connected, let M be the matroid shown in Figure 3. This matroid is connected and has $\{1, 2, 3, 4\}$ as a quad, yet it has no 4-element circuit containing the element e and hence certainly does not have a quad containing e.

We also note here that one cannot replace "pair" in Theorem 4.2 by "triple" since, for example, any three spokes in a wheel or a whirl do not all lie in a common circuit and hence do not all lie in a quad.

By Corollary 4.3 and Seymour's corresponding result for triads [8, (3.1)], when $k=3$ or 4, if a 3-connected matroid M has a k-element subset which is the intersection of a circuit and a cocircuit, then every pair of elements of M is in such a k-set. To see that this does not hold for all k, consider the matroid M shown in Figure 4. It is easy to check that M is 3-connected. Moreover, $\{2, 4, 6, 7, 8\}$ is both a circuit and a cocircuit of M. However, no 5-element subset of M which is the intersection of a circuit and a cocircuit contains the element 5.

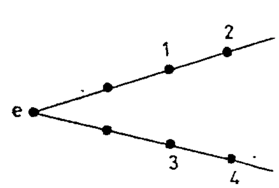

Fig. 3

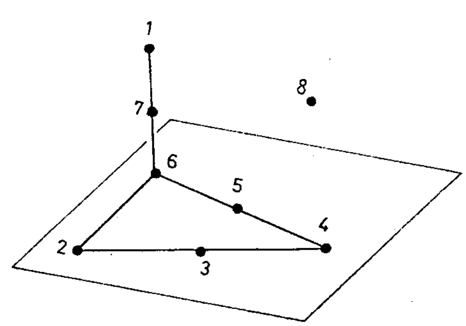

Fig. 4

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