

**CAPACITARY INEQUALITIES AND QUASILINEAR  
RICCATI TYPE EQUATIONS WITH CRITICAL OR  
SUPER-CRITICAL GROWTH**

NGUYEN CONG PHUC

ABSTRACT. We establish explicit criteria of solvability for the quasilinear Riccati type equation  $-\Delta_p u = |\nabla u|^q + \omega$  in a bounded  $C^1$  domain  $\Omega \subset \mathbb{R}^n$ ,  $n \geq 2$ . Here  $\Delta_p$ ,  $p > 1$ , is the  $p$ -Laplacian,  $q$  is critical  $q = p$  or super critical  $q > p$ , and the datum  $\omega$  is a measure. Our existence criteria are given in the form of potential theoretic or geometric (capacitary) estimates that are sharp when  $\omega$  is compactly supported in the ground domain  $\Omega$ . A key in our approach to this problem is capacity inequalities for certain nonlinear singular operators arising from the  $p$ -Laplacian.

DEPARTMENT OF MATHEMATICS, LOUISIANA STATE UNIVERSITY, 203 LOCKETT HALL,  
BATON ROUGE, LA 70803, USA.

*E-mail address:* pcnguyen@math.lsu.edu

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