SE Lie Theory Workshop 2018: Contributed Talks

Speaker: Hamza Alzaareer
Affiliation: Al-Zaytoonah University of Jordan

Title: *Mapping Lie Groups*

Abstract: We characterize the existence of Lie group structures on groups of the form $C^k(M, K)$, where M is a non-compact smooth manifold and K is a, possibly infinite-dimensional, Lie group. Motivated by introducing this new class of infinite-dimensional Lie groups, we obtain a new version of the Fundamental Theorem for Lie algebra-valued functions.

Speaker: Irfan Bagci
Affiliation: University of North Georgia

Title: *Weyl modules and Weyl functors for Lie superalgebras*

Abstract: Given an algebraically closed field k of characteristic zero, a Lie superalgebra $\mathfrak{g}$ over k and an associative, commutative k-algebra A with unit, a Lie superalgebra of the form $\mathfrak{g} \otimes A$ is known as a map superalgebra. Recently, in joint work with L. Calixto and T. Macedo, we defined Weyl functors, global and local Weyl modules for all map superalgebras where $\mathfrak{g}$ is a finite-dimensional simple Lie superalgebra not of type $\mathfrak{q}(n)$. Under certain conditions on the triangular decomposition of these Lie superalgebras we proved that global and local Weyl modules satisfy certain universal and tensor product decomposition properties. We also give necessary and sufficient conditions for local (resp. global) Weyl modules to be finite dimensional (resp. finitely generated). This talk will be a brief summary of these results.

Speaker: Seth Baldwin
Affiliation: University of North Carolina

Title: *Positivity in T-equivariant K-theory of Kac-Moody flag varieties*

Abstract: The cohomology ring of flag varieties has long been known to exibit positivity properties. One such property is that the structure constants of the Schubert basis with respect to the cup product are nonnegative. Further, Brion (2002) and Anderson-Griffeth-Miller (2011)

have shown that positivity extends to K-theory and T-equivariant K-theory, respectively. In this talk I will discuss recent work (joint with Shrawan Kumar) which generalizes these results to the case of Kac-Moody groups.

Speaker: Anne Dranowski
Affiliation: University of Toronto

Title: *Lusztig datum of an open MV cycle*

Abstract: In type A, equivariant volumes of quiver varieties $M(\lambda, \mu)$ satisfy the rational level 1 qKZ equation (Zinn-Justin, 2015). Under the Mirkovic-Vybornov isomorphism the volumes of irreducible components of M are identified with dual classes to Mirkovic-Vilonen cycles. I'll explain the combinatorics underlying the identification in the context of a conjecture involving a third player, Lusztig's semicanonical basis.

Speaker: Christopher Drupieski
Affiliation: DePaul University

Title: *Rank varieties for infinitesimal supergroup schemes*

Abstract: In this talk I will discuss recent work with Jonathan Kujawa investigating cohomological support varieties for infinitesimal supergroup schemes. This work extends previous results of Suslin, Friedlander, and Bendel for (ordinary) infinitesimal group schemes.

Speaker: Colin Hagemeyer
Affiliation: University of California, Davis

Title: *Presentations of Lie algebra Representation categories: Spiders and Generalized Confluence*

Abstract: We wish to investigate the (tensor) category of the finite dimensional representations of a semisimple Lie algebra (or quantum enveloping algebra). To do this we will restrict to a particular subcategory (of tensor products of fundamental representations which is big enough that we don't lose any information) and then give a presentation of that subcategory using generating objects, morphisms, and relators. However, not all presentations are created equal, so we will discuss a

nice property of certain presentations (or more precisely some corollaries) which is a generalization of confluence, and give some results for sl_4 and certain semisimple Lie algebras. No background knowledge beyond elementary semisimple Lie algebra representation theory will be assumed.

Speaker: Iva Halacheva
Affiliation: University of Melbourne

Title: *Higher Schur-Weyl duality and the periplectic Lie superalgebra*

Abstract: Classically, Schur-Weyl duality relates the $\mathfrak{gl}(n)$-endomorphisms of d tensor copies the standard representation to the symmetric group S(d). Arakawa and Suzuki's work extends this to more general representations, where S(d) is upgraded to the degenerate affine Hecke algebra. This framework was subsequently generalized by Ehrig and Stroppel to the Lie algebra $\mathfrak{so}(2n)$, where the Brauer algebras and affine VW algebras play an analogous role. We will discuss the Lie superalgebra $\mathfrak{p}(n)$ and the role the Brauer and affine VW superalgebras play in describing its representation theory. The main ingredient will be a surprising quadratic Casimir element constructed using both $\mathfrak{p}(n)$ and $\mathfrak{gl}(n|n)$.

Speaker: William Hardesty
Affiliation: Louisiana State University

Title: *On the modular Lusztig–Vogan bijection (joint with P. Achar and S. Riche)*

Abstract: Let $\mathbb{O}$ be a DVR such that its residue field, $\mathbb{F}$, is algebraically closed, let $\mathbb{K}$ be the algebraic closure of its field of fractions, and let G be a (split) simple algebraic group over $\mathbb{O}$. It is known that over both $\mathbb{K}$ and $\mathbb{F}$, there exist bijections between the set of dominant weights, $\mathbf{X}^+$, and the set of irreducible equivariant vector bundles on nilpotent orbits. These bijections are obtained by comparing two t-structures on the bounded derived category of equivariant coherent sheaves on the nilpotent cone. In the $\mathbb{K}$ case, this is known as the "Lusztig–Vogan bijection". We will refer to the $\mathbb{F}$-version as the "modular Lusztig–Vogan bijection". In this talk I will explain how these two constructions can be related by a certain base change technique. In general, the reductive quotient of the centralizer of an $\mathbb{F}$-nilpotent element is actually

disconnected. However, its module category still has the structure of a highest weight category, which can be naturally induced from the highest weight structure for its connected component. I will also give an overview of this construction, and the role it plays in relating the two bijections.

Speaker: Alexander Heaton
Affiliation: University of Wisconsin-Milwaukee

Title: *Graded multiplicity in harmonic polynomials from the Vinberg setting*

Abstract: We consider a family of examples falling into the following context: Let G be a connected reductive algebraic group over the complex numbers. A subgroup, K, of fixed points of a finite-order automorphism acts on the Lie algebra of G. Each eigenspace of the automorphism is a representation of K. Let V be one of the eigenspaces. We consider the harmonic polynomials on V as a representation of K, which is graded by homogeneous degree. Given any irreducible representation of K, we will see how its multiplicity in the harmonic polynomials is distributed among the various graded components. The results are described in terms of counting integral points on faces of a polyhedron. The multiplicity in each graded component is given by intersecting these faces with an expanding sequence of *shells*.

Speaker: Mee Seong Im
Affiliation: United States Military Academy

Title: *On the affine VW supercategory*

Abstract: A construction of the affine VW supercategory arose from our study of the representation theory of periplectic Lie superalgebras $\mathfrak{p}(n)$. Letting V to be a superspace with $\mathbb{Z}_2$-grading and M to be a $\mathfrak{p}(n)$-module, we construct a super version of the degenerate BMW algebra in the process of examining higher Schur-Weyl duality for the tensor product of M with finitely-many copies of V. I will discuss affine VW superalgebras (the signed version of the affine VW algebra and an affine version of the Brauer superalgebra) and their center, and the affine VW supercategory and its connection to Brauer supercategory. This is joint with M. Balagovic, Z. Daugherty, I. Entova, I. Halacheva, J. Hennig, G. Letzter, E. Norton, V. Serganova and C. Stroppel.

Speaker: Garrett Johnson
Affiliation: North Carolina Central University

Title: *Subprime solutions of the classical Yang-Baxter equation*

Abstract: We introduce a new family of r-matrices for the Lie algebra $\mathfrak{sl}(n)$ that lies in the Zariski boundary of the Belavin-Drinfeld space M of quasi-triangular solutions to the classical Yang-Baxter equation. In this setting M is a finite disjoint union of components; exactly $\phi(n)$ of these components are $SL(n)$-orbits of single points. These points are the generalized Cremmer-Gervais r-matrices $r(i,n)$ which are naturally indexed by pairs of positive coprime integers, i and n, with $i < n$. A conjecture of Gerstenhaber and Giaquinto states that the boundaries of the Cremmer-Gervais components contain r-matrices having maximal parabolic subalgebras $\mathfrak{p}(i,n) \subseteq \mathfrak{sl}(n)$ as carriers. We prove this conjecture in the cases when $n \equiv \pm 1 \pmod{i}$. The subprime linear functionals $f \in \mathfrak{p}(i,n)^*$ and the corresponding principal elements $H \in \mathfrak{p}(i,n)$ play important roles in our proof. Since the subprime functionals are Frobenius in the cases when $n \equiv \pm 1 \pmod{i}$, this partly explains our need to require these conditions on i and n. We conclude with a proof of the GG boundary conjecture in an unrelated case, namely when $(i,n) = (5,12)$.

Speaker: John Miller
Affiliation: Baylor University

Title: Syzygies and Pieri Maps

Abstract: The talk will include a brief overview of the fundamental problem of Classical Invariant Theory: finding generators and relations (syzygies) for rings of invariants and, more generally, for modules of covariants. For the general linear groups, this problem is partially answered by Weyls First and Second Fundamental Theorems for the rings of invariants of several vectors and co-vectors. Furthermore, the higher sygygies of these rings of invariants are given by the Lascoux resolution of determinantal ideals. Our work extends the results of Weyl and Lascoux to modules of covariants. Explicit descriptions of the minimal free resolutions that appear in this context and some interesting examples will be provided.

Speaker: Nham Ngo
Affiliation: University of North Georgia

Title: *On the complexities of Frobenius kernels and finite Chevalley groups*

Abstract: Let G be a simple algebraic group defined over an algebraically closed field k of prime characteristic p. For a positive integer r, let $F_r : G \to G$ be the r-th iteration of the Frobenius morphism, let $G_r = Ker(F_r)$ the r-th Frobenius kernel of G and $G(\mathbb{F}_{p^r}) = G^{F_r}$ the finite Chevalley group. In this talk, we will present our results on the complexity $c_{G_r}(M)$ of a finite dimensional G-module M over G_r and then introduce an inequality between $c_{G_r}(M)$ and $c_{G(\mathbb{F}_{p^r})}(M)$. This is joint work with Paul Levy and Klemen Sivic.

Speaker: Bach Nguyen
Affiliation: Louisiana State University

Title: *Quantum cluster algebras at roots of unity*

Abstract: As a noncommutative analog of cluster algebras, quantum cluster algebras were defined by Berenstein and Zelevinsky in 2005. Since then, such algebras have been an active research area with important applications in the study of canonical bases, combinatorics and representation theory. We extend this theory to the setting of roots of unity. As application, we compute the discriminant for these algebras in the case of quantum double Bruhat cells. This is a joint work with K. Trampel and M. Yakimov.

Speaker: Veronika Pedic
Affiliation: University of Zagreb, Croatia

Title: *Weight and Whittaker modules for Weyl vertex algebra*

Abstract: We construct weight and Whittaker modules for the Weyl algebra using vertex-algebraic techiniques. We discuss the connection of affine Kac-Moody Lie algebras to Whittaker and weight modules. We determine the fusion rules between irreducible weight modules for the Weyl algebra. (This is a preliminary report based on a joint work with D. Adamovic)

Speaker: John Gabriel Prudente Pelias
Affiliation: University of the Philippines, Diliman

Title: Some Aspects of the Linear Algebra and Geometry of the Quaternions

Abstract: We discuss matrix representations of quaternions and how quaternions relate with some familiar matrix groups. We also explore matrices with quaternionic entries and various related aspects: possible determinants of quaternionic matrices and how this can be used to approach the study of the complexification of the quaternions. Finally, we connect the quaternions with rotations and the Bloch sphere of quantum mechanics.

Speaker: Suchada Pongprasert
Affiliation: North Carolina State University

Title: $D_5^{(1)}$- *Geometric Crystal corresponding to the Dynkin spin node* $i = 5$ *and its ultra-discretization*

Abstract: Let $\mathfrak{g}$ be an affine Lie algebra with index set $I = \{0, 1, 2, \cdots, n\}$ and $\mathfrak{g}^L$ be its Langlands dual. It is conjectured that for each Dynkin node $i \in I \setminus \{0\}$ the affine Lie algebra $\geq$ has a positive geometric crystal whose ultra-discretization is isomorphic to the limit of certain coherent family of perfect crystals for $\mathfrak{g}^L$. In this paper we construct a positive geometric crystal $\mathcal{V}(D_5^{(1)})$ in the level zero fundamental spin $D_5^{(1)}$- module $W(\varpi_5)$. Then we define explicit 0-action on the level ℓ known $D_5^{(1)}$- perfect crystal $B^{5,l}$ and show that $\{B^{5,l}\}_{l\geq 1}$ is a coherent family of perfect crystals with limit $B^{5,\infty}$. Finally we show that the ultra-discretization of $\mathcal{V}(D_5^{(1)})$ is isomorphic to $B^{5,\infty}$ as crystals which proves the conjecture in this case.

Speaker: Venkatesh Rajendran
Affliliation: Indian Institute of Science, Bangalore, India

Title:*Maximal closed subroot systems of real affine root systems*

Abstract: We completely classify and give explicit descriptions of the maximal closed subroot systems of affine root systems. As an application we describe a procedure to get the classification of all regular subalgebras of affine Kac Moody algebras in terms of their (real) root systems.

Speaker: Krishanu Roy
Affiliation: The Institute of Mathematical Sciences, Chennai, India

Title: *Weyl orbits of overextended type π-systems in simply-laced Kac-Moody algebras*

Abstract: Given a symmetrizable Kac-Moody algebra $\mathfrak{g}$, a π-system of $\mathfrak{g}$ is a subset of its real roots such that pairwise differences are not roots. When $\mathfrak{g}$ is finite dimensional, Dynkin showed that linearly independent π–systems arise precisely as simple systems of regular semisimple subalgebras of $\mathfrak{g}$. He also computed the number of Weyl group orbits for each π-system in $\mathfrak{g}$. We prove if any symmetrizable Kac-Moody algebra $\mathfrak{g}$ admits a linearly independent π-system of affine type, then the number of Weyl orbits of π-systems of this type is necessarily infinite. We also prove if $\mathfrak{g}$ is simply-laced and the π-system is (simply-laced) of overextended type, then the number of Weyl group orbits is finite, and can in fact be obtained as a sum of the number of orbits of certain finite type π-systems inside finite root systems. This is a joint work with Lisa Carbone, K N Raghavan, Biswajit Ranisingh and Sanakaran Viswanath.

Speaker: Rahul Singh
Affiliation: Northeastern University

Title: *Spaltenstein Varieties and Orbital Varieties*

Abstract: An orbital variety is an irreducible component of the intersection of a nilpotent orbit closure with the set of upper triangular matrices. Each orbital variety is the image of the conormal variety Z_w of some Schubert variety X_w under the Springer map. If the Young diagram of the nilpotent orbit has two-columns, we may even choose a Schubert subvariety of a Grassmannian.

In this case, we construct a resolution of singularities for Z_w, and describe a system of equations defining Z_w set-theoretically. As a consequence, we obtain a system of equations defining for any two-column orbital set-theoretically. This also yields a combinatorial characterization of the inclusion poset of two-column orbital varieties.

Speaker: Michael Strayer
Affiliation: University of North Carolina

Title: *Minuscule representations of Kac-Moody algebras*

Abstract: The notion of "minuscule" representation of a semisimple Lie algebra has been formulated in terms of its highest weight. It can be seen that this approach does not produce any representations of the infinite dimensional Kac-Moody algebras. However, R. M. Green has explicitly constructed some "doubly infinite" representations of most affine Kac-Moody algebras using "full heap" colored posets. These representations are multiplicity free, their weight diagrams are "connected," and $(*)$ the action of every simple coroot on the weight basis is diagonal with eigenvalues in $\{-1, 0, 1\}$. We propose defining a "minuscule" representation of a Kac-Moody algebra to be one that has these three properties. Our talk presents the first stage in a program that seeks to classify all such representations of all simply laced Kac-Moody algebras. We revisit Green's construction, now considering arbitrary finite and locally finite colored posets uniformly. We obtain conditions on the poset coloring that are necessary as well as sufficient for the corresponding modules of the derived subalgebra to satisfy $(*)$. These posets are exactly the full heap posets found by Green, plus the finite minuscule posets. We anticipate that this result can be used to show that there are no minuscule representations of simply laced Kac-Moody algebras beyond those constructed by Green (and classified by Green and McGregor-Dorsey) and those for semisimple Lie algebras.

Speaker: Wan-Yu Tsai
Affiliation: Academia Sinica, Taiwan

Title: *The orbit philosophy for Spin groups of type D*

Abstract: In this talk, we provide a comparison between the K-structure of unipotent representations and regular functions of bundles on nilpotent orbits for complex and real groups of type D. More precisely, we provide a list of genuine unipotent representations for a Spin group; separately we compute the K-spectra of the regular functions on certain small nilpotent orbits, and then match them with the K-types of the genuine unipotent representations. The results provide instances for the orbit philosophy. This is joint work with Dan Barbasch.

Speaker: Arik Wilbert
Affiliation: University of Melbourne

Title: *Exotic Springer fibers and two-boundary Temperley-Lieb algebras*

Abstract: In this talk we will study the geometry and topology of a certain family of exotic Springer fibers. These algebraic varieties appear as the fibers under a resolution of singularities of the exotic nilpotent cone which plays a prominent role in Kato's work on the representation theory of multiparameter Hecke algebras of type C. We describe our results in terms of the combinatorics of the two-boundary Temperley-Lieb algebra. This relates the exotic Springer fibers to interesting categorification problems arising in low-dimensional topology.

Speaker: Ziqing Xiang
Affiliation: University of Georgia

Title: *Support varieties for Hecke algebras*

Abstract: In this talk, I will discuss a theory of support varieties for Hecke algebras which detects natural homological properties such as the complexity of modules. The theory has a canonical description in an affine space where computations are tractable. The ideas involve the interplay with the computation of the cohomology ring due to Benson, Erdmann and Mikaelian, the theory of vertices due to Dipper and Du, and branching results for cohomology by Hemmer and Nakano. Calculations of support varieties and vertices are presented for permutation, Young and classes of Specht modules. Furthermore, generalization to other Hecke algebras for other classical groups will be discussed. This is joint work with Dan Nakano.

Speaker: Yue Zhao
Affiliation: University of California, Davis

Title: *Irreducible representations of Cherednik algebras and periodic skew diagrams*

Abstract: This talk is based on a joint work of Takeshi Suzuki and Monica Vazirani. There is a combinatorial description of irreducible representations of Cherednik algebras with certain properties, which

gives a correspondence between irreducible representations of Cherednik algebras and periodic skew diagrams.

Speaker: Jieru Zhu
Affiliation: University of Oklahoma

Title: *Two Boundary Centralizer Algebras for* $\mathfrak{gl}(n|m)$

Abstract: The degenerate two boundary Hecke algebra $\mathcal{H}_d$ is generated by the symmetric group on d letters and polynomial rings subject to further relations. It acts on the tensor space $M \otimes N \otimes V^{\otimes d}$, where M and N are irreducible polynomial representations of the Lie superalgebra $\mathfrak{gl}(n|m)$ whose highest weights are represented by rectangular Young diagrams, and this action commutes with that of $\mathfrak{gl}(n|m)$. As a module for the centralizer of $\mathfrak{gl}(n|m)$, $M \otimes N \otimes V^{\otimes d}$ decomposes into irreducible modules labeled by hook Young diagrams, and a basis is given via Young tableaux where the polynomial generators act by explicit combinatorial eigenvalues. These modules remain irreducible when restricted to the action of the Hecke algebra, and provide a class of irreducible representations. This construction generalizes results in the $\mathfrak{gl}(n)$ setting by Zajj Daugherty (2010).