Southeastern Lie Theory Workshop XI

Geometric and Categorical Representation Theory

Louisiana State University • Baton Rouge, LA • May 13–15, 2019

Main Schedule

The main talks will be held in the Design Building Room 103. Contributed talks (Monday afternoon) will be held in Lockett Hall.

All coffee breaks except the Monday afternoon coffee break will also be in the Design Building.

Conference participants are welcome to use Lockett Hall Rooms 232, 243, and 285 at any time during the conference, except during the scheduled times for contributed talks. There is also lounge on the 3rd floor of Lockett Hall, Room 321.

	Mon., May 13	Tues., May 14	Wed., May 15
8:30am	Registration	Yun 1	Lanini
9:00am	Nadler 1		
9:30am		Coffee break	Coffee break
10:00am	Coffee break	Nadler 2	Yun 2
10:30am	Arinkin		
11:00am			
11:30am	Lunch break	Cliff	Lusztig
12:00pm		Lunch break	
12:30pm			
1:00pm			
1:30pm	Contributed talks		
2:00pm			
2:30pm		Elias	
3:00pm	Coffee break		
3:30pm	Contributed talks	Coffee break	
4:00pm		Ben-Zvi	
4:30pm			

Monday, May 13

9:00–10:00 **David Nadler**, *Degenerating eigenvalues*

Characteristic polynomials fiber Lie algebras over their eigenvalues. Springer theory in its many forms then studies how generic fibers degenerate to special fibers. But one can also degenerate the space of eigenvalues itself. I will explain how this can be implemented and then applied to nilpotent cones, in particular the Kostant-Sekiguchi correspondence (based on joint work with Tsao-Hsien Chen).

10:30–11:30 **Dima Arinkin**, *Cameral covers and Higgs bundles: additive, multiplicative, and elliptic*

Higgs bundles are natural geometric objects that have been studied from many different directions. One of the key tools is the Hitchin fibration, which is the geometric version of a fundamental idea from linear algebra: the data (Higgs bundle) is split into spectral data ('eigenvalues') and spacial data ('eigenspaces'). A further development of this idea is the theory of cameral covers due to R.Donagi and D.Gaitsgory.

In my talk, I will extend the theory of cameral covers in two directions: to Higgs fields that need not be regular, and to different kinds of Higgs bundles, such as 'group-valued' Higgs bundles. This allows us to treat, in a uniform way, various 'Higgs bundle-like' objects, such as usual or group-valued Higgs bundles, semistable bundles on an elliptic curve, and perhaps even the space of regular connections on a punctured disk.

Tuesday, May 14

8:30– 9:30 **Zhiwei Yun**, *Endoscopic equivalence for Hecke categories*

Hecke categories are geometric incarnations of Hecke algebras, and they play an important role in the classification of irreducible representations of finite groups of Lie type. We consider a version of the Hecke category for the reductive group G with prescribed but arbitrary monodromy under the left and right actions of the maximal torus. We show that this monoidal category can essentially be identified with the usual Hecke category (with trivial torus monodromy) for an endoscopic group H of G, which is a reductive group of smaller dimension sharing a maximal torus with G but is not necessarily a subgroup of G. The main part of the proof is to relate the Hecke category with arbitrary monodromy to Soergel bimodules. This is joint work with G.Lusztig.

10:00–11:00 **David Nadler**, *Degenerating eigenvalues*

11:15–12:15 **Emily Cliff**, *Chiral algebras, factorization algebras, and Borcherds' "singular commutative rings" approach to vertex algebras*

In the late 1990s, Borcherds gave an alternate definition of some vertex algebras as "singular commutative rings" in a category of functors depending on some input data (A, H, S). He proved that for a certain choice of A, H, and S, the singular commutative rings he defines are indeed examples of vertex algebras. In this talk I will explain how we can vary this input data to produce categories of chiral algebras and factorization algebras (in the sense of Beilinson–Drinfeld) over certain complex curves X. We'll discuss the failure of these constructions to give equivalences of categories, and obstructions to extending this approach to more general varieties X. I will not assume background in vertex algebras, factorization algebras, or chiral algebras.

2:30– 3:30 **Ben Elias**, *Gaitsgory's central complexes and the Gorsky-Negut-Rasmussen conjecture*

When studying the representation theory of the Hecke algebra of the symmetric group, there is a large commutative subalgebra, the Jucys-Murphy subalgebra, which plays a crucial role. Analogous to the Cartan subalgebra of a semisimple Lie algebra, simultaneously diagonalizing the Jucys-Murphy subalgebra is an immensely powerful tool. There is a categorification of this story, where the Hecke algebra is replaced by the triangulated Hecke category (whose objects are complexes), and the Jucys-Murphy subalgebra by a triangulated subcategory. The simultaneous diagonalization of the Jucys-Murphy complexes was recently achieved in joint work with Hogancamp.

An astounding recent conjecture of Gorsky-Negut-Rasmussen states that the Jucys-Murphy subcategory is equivalent to coherent sheaves on (roughly) the flag Hilbert scheme of points on the plane, and the center of the Hecke category is equivalent to coherent sheaves on (roughly) the ordinary Hilbert scheme. They reduce this conjecture to finding, inside the Hecke category, the central complex $\mathcal{E}_1$ which corresponds to the first elementary symmetric polynomial in the Jucys-Murphy operators, and studying its properties.

Meanwhile, in the 90s, Gaitsgory constructed a monoidal functor, similar in spirit to geometric Satake, which (after rephrasing) goes from representations of $\mathfrak{gl}_n$ to the center of the extended affine Hecke category. Just as a representation has a splitting into weight spaces, Gaitsgory's central complexes have a filtration by Wakimoto complexes. Recently, I have given an explicit construction of the image $\mathcal{V}$ of the standard representation. (There is an independent construction of $\mathcal{V}$ in another context due to Achar-Rider.) I also conjecture that there is a flattening functor from the extended affine Hecke category to the finite Hecke category, which sends Wakimoto complexes to Jucys-Murphy complexes, and sends $\mathcal{V}$ to $\mathcal{E}_1$.

In this talk I try to give an overview of the many moving parts in this picture, and I will try to describe the explicit construction of $\mathcal{V}$.

4:00– 5:00 **David Ben-Zvi**, *Coherent Affine Springer Theory*

A celebrated result of Kazhdan and Lusztig identifies the affine Hecke algebras H_{aff} with the equivariant K-group of the Steinberg variety. They applied this identification to establish the local Langlands classification of irreducible H_{aff}-modules. I will describe a families version of this result, realizing the derived category of H_{aff}-modules as a category of coherent sheaves on the stack of Langlands parameters. This is motivated by recent developments in the local Langlands correspondence. The proof uses ideas from derived algebraic geometry, and in particular realizes H_{aff} itself as a convolution algebra of volume forms on the equivariant derived loop space of the Steinberg variety. Based on joint work with Harrison Chen, David Helm and David Nadler.

Wednesday, May 15

8:30– 9:30 **Martina Lanini**, *Twisted quadratic foldings of root systems*

In this talk I will discuss joint work with Kirill Zainoulline about twisted foldings of root systems. This construction generalises usual involutive foldings corresponding to automorphisms of Dynkin diagrams. Our motivating example is the non-split folding of the root system of type E_8 which gives rise to the root system of type H_4, appeared already in the Eighties in work of Lusztig. Using moment graph techniques we show that twisted quadratic foldings can be applied to obtain information about equivariant cohomology.

10:00–11:00 **Zhiwei Yun**, *Endoscopic equivalence for Hecke categories*

11:15–12:15 **George Lusztig**, *A new basis for the Grothendieck group of unipotent representations*

Let U be the set of isomorphism classes of unipotent representations of a finite Chevalley group. Let $[U]$ be the complex vector space with basis U. We define a new basis $\{\hat{b}; b \in U\}$ of $[U]$ indexed by U. Each new basis element $\hat{b}$ is a linear combination of old basis elements with coefficients natural numbers; the coefficient of b is 1.

Contributed talks

The contributed talks will be held in two parallel sessions in Lockett Hall Rooms 232 and 243. The Monday afternoon coffee break will take place in the 3rd floor lounge of Lockett Hall (Room 321).

	Lockett Hall 232	Lockett Hall 243
1:30pm	Lai	
2:00pm	Xiang	Cowie
2:30pm	Trampel	Im
3:00pm	Coffee break	
3:30pm	Jeralds	Vashaw
4:00pm	Murray	Brown
4:30pm	Casper	Colarusso

Parallel Session 1 (Lockett Hall 232)

1:30– 1:50 **Chun-Ju Lai**, *A coordinate coalgebra realization of q-Schur duality of type B*

We investigate the q-Schur algebras of type B that were constructed earlier using coideal subalgebras for the quantum groups of type A. We present a coordinate coalgebra construction that allows us to realize these q-Schur algebras as the duals of the d-th graded components of certain graded coalgebras. Such a realization allows us to relate the representation theory of the type B q-Schur algebras with the representation theory of the associated quantum symmetric pairs.

2:00– 2:20 **Ziqing Xiang**, *Isomorphism theorem between q-Schur algebras of type B and type A*

This talk focuses on Schur algebras of type B with two parameters q and Q. I will show that for arbitrary q and generic Q, the type B Schur algebras are isomorphic to direct sums of tensor products of type A Schur algberas. This enables us to address the questions of cellularity, quasi-hereditariness and representation type of the type B Schur algebras. This isjoint work with Chun-Ju Lai and Daniel Nakano.

2:30– 2:50 **Kurt Trampel**, *Quantum cluster algebras at roots of unity*

A general construction is built for quantum cluster algebras at roots of unity. For such an algebra, we construct a canonical central subalgebra that is isomorphic to the classic cluster algebra with the same exchange matrix. This recovers the central subalgebras of quantum groups at roots of unity used by De Concini-Kac-Procesi in special cases. We take first steps to studying the representation theory of these quantum cluster algebras. In particular, we prove a general theorem on the form of the discriminants of these algebras. Applicable examples are quantum Schubert cells and Heisenberg doubles.

3:30– 3:50 **Sam Jeralds**, *Root Components and Tensor Product Decompositions for Affine Lie Algebras*

In his proof of the surjectivity of the Gaussian map for flag varieties, Kumar exhibited the existence of “root components” in the decomposition of tensor products of irreducible highest weight modules. In this talk, we will extend his construction to the case of affine Lie algebras and discuss analogous results for their highest weight modules.

4:00– 4:20 **Kayla Murray**, *Chari-Venkatesh Modules*

In this talk, we will discuss graded representations of the current algebra associated to the Lie algebra sl_2. The representations we will focus on are Chari-Venkatesh modules, which depend on a partition. This family of modules includes both Demazure modules and local Weyl modules.

4:30– 4:50 **William Riley Casper**, *The Matrix Bochner Problem and Representation Theory*

We present a solution of the matrix Bochner problem, a long-standing open problem in the theory of orthogonal polynomials, with applications to diverse areas of research including representation theory, random matrices, spectral theory, and integrable systems. Our solution is based on ideas applied by Krichever, Mumford, Wilson and others, wherein the algebraic structure of an algebra of differential operators influences the values of the operators in the algebra. By using a similar idea, we convert the matrix Bochner problem to one about non-commutative algebras of GK dimension 1 which are module finite over their centers. Then the problem is resolved using the representation theory of these algebras.

Parallel Session 2 (Lockett Hall 243)

2:00– 2:20 **Emily Cowie**, *The Hilbert series and a-invariant of circle invariants*

Given a complex reductive group, we can construct the algebra of G-invariant polynomials. The Hilbert series derived from this graded algebra contains important information about the algebra, and as such is used in constructive invariant theory as well as to determine which representations of G have Groenstein invariant rings. Though the methods of computing the Hilbert series are relatively assessable, deriving the Hilbert series can be computationally expensive. In this talk, I will present formulas for the Hilbert series as well as the first four coefficients of the Laurent expansion in terms of partial Schur polynomials.

2:30– 2:50 **Mee Seong Im**, *The geometry of parabolic Hamiltonian reduction*

In the construction of Hamiltonian reductions in symplectic geometry, interesting and rich connections to Hilbert schemes, Calogero-Moser spaces, and rational spherical Cherednik algebras have emerged over the last two decades. A parabolic analogue of the classical general linear group construction (realized after a reduction from the cotangent bundle of enhanced partial Grothendieck-Springer resolutions) potentially opens doors for its connections to isospectral Hilbert schemes, partial flag Hilbert schemes, and other algebraic varieties, that are important to geometric representation theory, algebraic combinatorics, and quantum topology.

Our construction can also be realized by certain quiver (partial) flag varieties, appearing in the geometric interplay in quiver Hecke algebras that categorify quantum groups.

I will discuss a parabolic analogue of the cotangent bundle of the extended general linear Lie algebra, discussing the complete intersection of the zero fiber of a moment map (as conjectured by Thomas Nevins), an enumeration of the irreducible components, and a parabolic analog of an almost-commuting scheme appearing in the study of Calogero-Moser systems.

This is joint with Travis Scrimshaw.

3:30– 3:50 **Kent Vashaw**, *Prime Spectra of Abelian 2-Categories and Categorification of Richardson Varieties*

We describe a general theory of the prime, semiprime, completely prime, and primitive spectra of an abelian 2-category, providing a noncommutative version of Balmers prime spectrum of a tensor triangulated category. The prime ideals of an abelian 2-category can be described in terms of containment conditions of either thick or Serre ideals of the 2-category. The Serre prime spectrum of a 2-category is linked to the set of Serre primes of its Grothendieck ring. We construct a categorification of the quantized coordinate rings of open Richardson varieties for symmetric Kac-Moody groups, by constructing Serre completely prime ideals of monoidal categories of modules of the KLR algebras, and by taking Serre quotients with respect to them. This is a joint work with Milen Yakimov.

4:00– 4:20 **Adam Brown**, *The geometric foundation of Arakawa-Suzuki functors*

Arakawa-Suzuki functors generalize Schur-Weyl duality by mapping highest weight modules over a complex Lie algebra to modules over a graded affine Hecke algebra. Many of the important properties exhibited by these functors are based on geometric relationships between Schubert varieties and graded nilpotent classes. In this talk we will discuss how these geometric results relate Whittaker modules of semisimple Lie algebras with representations of graded affine Hecke algebras.

4:30– 4:50 **Mark Colarusso**, *Orbits of multiplicity free spherical subgroups on the flag variety*

Let G be an algebraic group over $\mathbb{C}$, and let $K = G^{\theta}$ be a symmetric subgroup. The theory of K-orbits on the flag variety G/B of G was studied extensively by Richardson and Springer. The geometry of these orbits plays a central role in the construction of classical Harish-Chadra modules via the Beilinson-Bernstein correspondence. Much less is known about the theory of K-orbits on G/B when K is a *spherical* subgroup of G. In this talk, we describe geometric and combinatorial properties of the K-action on G/B, when (G, K) is a multiplicity free spherical pair. Using an analogue of the Beilinson-Bernstein correspondence, the geometry of these orbits can be used to construct a certain category of generalized Harish-Chandra modules related to the category of Gelfand-Zeitlin modules which quantize the Gelfand-Zeitlin integrable system. This is joint work with Sam Evens.