# The 15th Southeastern Lie Theory Workshop Program College of Charleston

# Cornelius Pillen Day on Friday, May 2nd

	Saturday morning Room: HWWE 213	Sunday morning Room: HWWE 213	
	Bălibanu (9:00–9:50am)	Matherne $(9:00-9:50am)$	
	Coffee break (10:00–10:30am)	Coffee break (10:00–10:30am)	
	Harman (10:30–11:20am)	Kanade (10:30–11:20am)	
	Panel Discussion (11:30am–12:30pm)		
Friday afternoon Room: HWWE 213	Saturday afternoon Contributed talks		
Sobaje (2:00–2:50pm)	Parallel Sessions (2:30–4:00pm)		
Coffee break (3:00-3:30pm)	Coffee break (4:00–4:30pm)		
Wilbert (3:30–4:20pm)	Parallel Sessions (4:30–5:30pm)		
Drupieski (4:30–5:20pm)			

All talks are in the building Harbor Walk located at 360 Concord Street, Charleston SC 29401

Panel Discussion Research at non-R1 Institutions, Chaired by Liz Jurisich.

	Session 1 HWWE 211	Session 2 HWWE 305	Session 3 HWWE 307
2:30–2:50pm	Hamil	Zhu	Malaney
3:00-3:20pm	Khovanov	Addabbo	Hu
3:30–3:50pm	Wu	Chen	Zveryk
4:00-4:30pm	Coffee and tea break		
4:30–4:50pm	Zhou	Nakano	Upadhyay
5:00-5:20pm	Im	Loos	Larson

# Contributed Talks

All contributed talks are in the format of 20-minute presentations plus 5-minute Q&A.

# **Program:** Titles and Abstracts of Invited Addresses

**Ana Bălibanu** An analogue of Whittaker reduction for group-valued moment maps Louisiana State University, Email: ana@math.lsu.edu

Abstract: Let G be a semisimple complex group and let M be a Hamiltonian G-space. Whittaker reduction is a type of Hamiltonian reduction along Slodowy slices that encodes the Poisson geometry of M in the direction transverse to the action of G. We construct a multiplicative analogue of this reduction in the setting of Poisson-Lie groups, where the moment map takes values in the group G (rather than in the dual of its Lie algebra). Whittaker reduction then occurs along a class of transversal slices to unipotent orbits in G which generalize the Steinberg crosssection and are indexed by conjugacy classes in the Weyl group.

# Christopher Drupieski Some Lie (super)algebras generated by reflections

## DePaul University, Email: c.drupieski@depaul.edu

Abstract: In 2007, motivated by questions from the representation theory of the braid group, Ivan Marin computed the structure of the Lie algebra generated by the transpositions in the group algebra of the symmetric group. In this talk I'll describe Marin's results, and then describe joint work with Jonathan Kujawa investigating the analogous questions for Weyl groups of types B and D (and beyond...?), as well results on the Lie superalgebras generated by the same sets of elements.

# Nate Harman Interpolation Categories for Classical Groups

# University of Georgia, Email: nharman@uga.edu

Abstract: Deligne defined a family of categories  $\operatorname{Rep}(S_t)$  for arbitrary complex numbers t which "interpolate" the representation theory of symmetric groups  $S_n$ . These have become important examples in the theory of symmetric tensor categories, and there has been a lot of work generalizing their construction to a number of other families of groups. Notably missing though were interpolation families for the symplectic, orthogonal, and unitary groups over a finite field — these were long believed to exist, but none of the previous constructions were actually able to construct them. I will discuss recent work with Andrew Snowden where we resolve this issue and are able to construct these categories using oligomorphic groups.

# Shashank Kanade Torus links and characters of vertex operator algebras

#### University of Denver, Email: shashank.kanade@du.edu

*Abstract:* In this talk, I will explain some of my recent results relating colored invariants of torus links with characters of various vertex operator algebras. This relationship relies on conjectural asymptotic weight and tensor product multiplicities for finite-dimensional irreducible modules of finite-dimensional simple Lie algebras. Work on these conjectures is ongoing with Arvind Ayyer.

# **Jacob Matherne** Kazhdan–Lusztig functions and Chow functions of partially ordered sets North Carolina State University, Email: jpmather@ncsu.edu

Abstract: Three decades ago, Stanley and Brenti initiated the study of the Kazhdan–Lusztig–Stanley (KLS) functions. Roughly, they showed how to associate a KLS function to any poset, thus putting on common ground three important examples: the classical Kazhdan–Lusztig polynomials of Coxeter groups, the matroidal Kazhdan–Lusztig polynomials, and the toric g-polynomials of polytopes. In this talk, we will recall the KLS story with a focus on these three important examples.

We will also develop a theory that parallels the KLS theory by associating a so-called Chow function to any poset. In the three respective examples above, the Chow function enumerates paths in the Bruhat graph according to a descent-like statistic, is the Hilbert series of the Chow ring of a matroid (hence the name "Chow function"), and is the h-polynomial of the barycentric subdivision of the poset. We will explore the relation between the KLS and Chow functions, and illustrate how one can build upon intuition from one setting (say, polytopes) and use it in another (say, Coxeter groups). Based on joint works with Luis Ferroni and Lorenzo Vecchi, as well as with Tom Braden, June Huh, Nicholas Proudfoot, and Botong Wang.

# Paul Sobaje Tilting Modules and Filtrations For Algebraic Groups

# Georgia Southern University, Email: psobaje@georgiasouthern.edu

Abstract: I will survey the status of several interconnected conjectures pertaining to tilting modules of a reductive group G in prime characteristic. These conjectures were made by Donkin 35 years ago, and deal with properties of G-modules that are unique to the modular setting (in particular, they are not interesting for quantum groups at roots of unity). This is joint work with Chris Bendel, Dan Nakano, and Cornelius Pillen.

# Arik Wilbert Two-row Delta Springer varieties

# University of South Alabama, Email: wilbert@southalabama.edu

Abstract: I will discuss the geometry and topology of a certain family of so-called Delta Springer varieties from an explicit, diagrammatic point of view. These singular varieties were introduced by Griffin-Levinson-Woo in 2021 in order to give a geometric realization of an expression that appears in the t = 0 case of the Delta conjecture of Haglund, Remmel, and Wilson. In the two-row case, Delta Springer varieties generalize both ordinary Springer fibers as well as Kato's exotic Springer fibers. Moreover, the homology of two-row Delta Springer varieties has a diagrammatic description and can be equipped with an action of the degenerate affine Hecke algebra. This recovers and upgrades the action of the symmetric group obtained by Griffin-Levinson-Woo and yields a skein theoretic description of said action. This is joint work with A. Lacabanne and P. Vaz.

# **Program:** Titles and Abstracts of Contributed Talks

**Darlayne Addabbo** Modularity of Vertex Operator Algebra Correlators with Zero Modes SUNY Polytechnic Institute Email: addabbd@sunypoly.edu

*Abstract:* In this talk, we will discuss modular transformation properties of VOA correlators that have zero modes inserted. In particular, we will discuss recursion relations for these correlators and describe how to use these recursions to establish the modular transformation properties. This talk is based on joint work with Christoph A. Keller.

# Hong Chen Differential Equations for Macdonald's Hypergeometric Functions

Rutgers University Email: hc813@math.rutgers.edu

Abstract: In his widely circulated 1980s manuscript (now available as [arXiv:1309.4568]), Macdonald introduced hypergeometric functions with a parameter  $\alpha$  (the Jack parameter) and posed foundational questions about their properties. Some of these questions have since been addressed for instance, Brennecken and Rösler (2023) studied the Dunkl–Laplace transform. In this work, we present a general construction of differential operators  ${}_pD_q$  that characterize Macdonald's hypergeometric functions  ${}_pF_q$  for arbitrary p and q.

Matthew Hamil Localizing subcategories for Lie superalgebra representations

#### University of Georgia Email: mhh14981@uga.edu

Abstract: Let  $\mathfrak{g} = \mathfrak{g}_{\bar{0}} \oplus \mathfrak{g}_{\bar{1}}$  be a Type I classical Lie superalgebra. One can consider the category  $\mathcal{C}_{(\mathfrak{g},\mathfrak{g}_{\bar{0}})}$  of  $\mathfrak{g}$ -modules which are finitely semisimple as modules over  $\mathfrak{g}_{\bar{0}}$ . On the other hand, Boe, Kujawa, and Nakano introduced interesting subsuperalgebras  $\mathfrak{z} \leq \mathfrak{g}$ , called detecting subalgebras, which determine the relative  $(\mathfrak{g},\mathfrak{g}_{\bar{0}})$ -cohomology. The categories  $\mathcal{C}_{(\mathfrak{g},\mathfrak{g}_{\bar{0}})}$  and  $\mathcal{C}_{(\mathfrak{z},\mathfrak{z}_{\bar{0}})}$  are Frobenius, so one can form their stable categories and which are tensor triangulated categories. This raises natural questions regarding the tensor ideal localizing subcategories, which we will address.

Mengwei Hu On certain Lagrangian subvarieties in minimal resolutions of Kleinian singularities Yale University Email: m.hu@yale.edu

Abstract: Kleinian singularities are remarkable singular affine surfaces. They arise as quotients of  $\mathbb{C}^2$  by finite subgroups of  $SL_2(\mathbb{C})$ . The exceptional loci in the minimal resolutions of Kleinian singularities are in 1-to-1 correspondence with the simply-laced Dynkin diagrams. In this talk, I will introduce certain singular Lagrangian subvarieties in the minimal resolutions of Kleinian singularities that are related to the classification of irreducible Harish-Chandra ( $\mathfrak{g}, K$ )-modules annihilated by certain unipotent ideals. The irreducible components of these singular Lagrangian subvarieties are  $\mathbb{P}^1$ 's and  $\mathbb{A}^1$ 's. I will describe how they intersect with each other through the realization of Kleinian singularities as Nakajima quiver varieties.

# Mee Seong Im One-dimensional cobordisms with defects and pseudocharacters

Johns Hopkins University Email: meeseong@jhu.edu

*Abstract:* Pseudocharacters are an essential tool in modern number theory. I will explain how group characters and pseudocharacters appear in topological theory and TQFTs in one dimension with defects. This is joint with M. Khovanov and V. Ostrik.

## Dan Nakano On the Hochschild Cohomology for Frobenius Kernels

University of Georgia Email: nakano@uga.edu

Abstract: In this talk, we will investigate the structure of the Hochschild cohomology for Frobenius kernels. After establishing some fundamental constructions using spectral sequences a complete description of the G-algebra structure of the Hochschild cohomology for the first Frobenius kernel  $G_1$  where  $G = SL_2$  is given. This computation heavily relies on the calculation of the adjoint action on the restricted enveloping algebra.

This is joint work with Tekin Karadag.

# Mikhail Khovanov Foams in representation theory

Johns Hopkins University Email: khovanov@jhu.edu

Abstract: We review the role and applications of foams in representation theory and categorification.

# Scott Larson On the Bala–Carter theory for quasireductive supergroup schemes

University of Georgia Email: scott.larson@uga.edu

*Abstract:* In joint work with Daniel K. Nakano, we use structure theory of Lie superalgebras to classify nilpotent orbits for quasireductive supergroup schemes. In the case where the quasireductive supergroup scheme corresponds to a connected complex reductive group, our theorem reduces to the classical Bala–Carter theorem.

#### Gabriel Loos On the Relative Cohomology for Algebraic Groups

# University of Georgia Email: gtl94940@uga.edu

Abstract: Let G be an algebraic group over a field k. In 1961, Hochschild showed how one can define Ext(M, N), and consequently the cohomology groups, for G and G-modules N and M. Afterwards, in 1965, Kimura showed that one can generalize this to get relative cohomology for algebraic groups. The cohomology groups play an important role in understanding the representation theory of G, but the role of relative cohomology has been left untouched. In this talk, I will expand upon the work of Kimura to get many generalizing results. In particular, I will classify relative injective modules, define a relative right derived functor, state when there exists a relative Grothendieck Spectral Sequence, and give some motivating examples of when relative cohomology is an interesting invariant.

### Alberto San Miguel Malaney Partial Resolutions of Affine Symplectic Singularities

# University of Georgia Email: as28174@uga.edu

Abstract: Symplectic singularities are a generalization of symplectic manifolds that have a symplectic form on the smooth locus but allow for certain well-behaved singularities. They have a strong relationship to representation theory and include nilpotent cones of semisimple Lie algebras, quiver varieties, and Kleinian singularities. In this talk we will explore crepant partial resolutions of symplectic singularities, studying their birational geometry, deformation theory, and Springer theory. In particular, we will review the definition of the Namikawa Weyl group for conical affine symplectic singularities and define a generalization for their partial resolutions.

**Aparna Upadhyay** Tensor powers of some symmetric group representations University of South Alabama Email: aupadhyay@southalabama.edu Abstract: The decomposition of tensor products of representations are fundamental in representation theory. In this talk we will analyze the asymptotic behavior of the decomposition of tensor powers of certain modular representations of the symmetric group.

# **Haihan Wu** SL(4) foams and degree-6 vertex

# Johns Hopkins University Email: hwu125@jhu.edu

*Abstract:* Khovanov categorified the Jones polynomial, which started a trend of categorifications of quantum invariants. One way to categorify the type A link invariants is through foams, 2-dimensional cellular complexes with facets of different thicknesses.

Inspired by the recent development on rotational-invariant SL(4) web basis by Gaetz-Pechenik-Pfannerer-Striker-Swanson, we modify the foam construction given by Robert-Wagner, introducing higher singularities.

In this talk, I will review background material on webs and foams and talk about the modified SL(4) foam construction. If time permits, I will also mention its connection to SL(4) web algebra and SL(4) affine building. This talk is based on upcoming joint work with Jernej Grlj, Mikhail Khovanov, and Melissa Zhang.

# Fan Zhou Categorifying the Jacobi-Trudi identity

# $Columbia \ University \quad Email: \ fz 2326 @columbia.edu$

*Abstract:* We categorify the Jacobi-Trudi identity by considering certain quotients of cyclotomic KLR algebras.

**Songhao Zhu** Growth of root multiplicities along imaginary root strings in Kac–Moody algebras Georgia Institute of Technology Email: szhu95@gatech.edu

Abstract: In a Kac–Moody algebra, every real root string is finite and has a constant multiplicity of one. In this work, we present how the multiplicities grow along imaginary root strings in symmetrizable Kac–Moody algebras. In particular, we show that when the imaginary root of the direction of a root string has a negative norm, then the growth is at least exponential, while when it has zero norm, then the growth is either bounded or at least faster than any polynomial.

# Vladyslav Zveryk Cohomological stability of quiver varieties

#### Yale University Email: vladyslav.zveryk@yale.edu

Abstract: Quiver varieties provide a bridge between the representation theory of quivers and the structure of the associated Kac–Moody Lie algebras. For instance, one could recover representations of Kac-Moody Lie algebras as cohomologies of Nakajima quiver varieties, and their root space multiplicities are encoded by cohomologies of another type of quiver varieties. In this work in progress, we observe a stabilization of cohomologies of quiver varieties when we add a given imaginary root to a fixed dimension vector a sufficient number of times. I will present formulas for the dimensions of stabilized cohomologies and will deduce certain bounds on root space multiplicities of Kac-Moody Lie algebras from them.