

18.024–ESG Exam 1

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1. Below are listed several linear subspaces of \mathbb{R}^n , each with a set of vectors. For each set of vectors, decide whether it is linearly independent, and decide whether it spans the given subspace. Finally, find the dimension of the subspace. Answers alone will suffice; you do not need to give any justification.

Linear subspace	Vectors	Linearly independent?	Spanning set?	Dimension
$\{(x_1, x_2, x_3) \mid x_1 + x_2 + x_3 = 0\} \subset \mathbb{R}^3$	$(1, -1, 0)$ $(0, 1, -1)$			
$\{(x_1, x_1, x_1)\} \subset \mathbb{R}^3$	$(1, 1, 1)$ $(-1, -1, -1)$			
\mathbb{R}^4	$(1, 1, 0, 0)$ $(0, 1, 1, 0)$ $(0, 0, 1, 1)$			

2. Solve the following system of equations by Gauss-Jordan elimination.

$$\begin{aligned} x + y + 3z &= 5 \\ 2x - y + 4z &= 11 \\ -y + z &= 3. \end{aligned}$$

3. Find a parametric equation for the 2-plane going through the points $(6, 9, 0, 2)$, $(6, 7, 0, 4)$, and $(3, 6, 2, 1)$ in \mathbb{R}^4 .
4. Let $\mathbf{a}, \mathbf{b} \in \mathbb{R}^3$.
- (a) Under what conditions is it true that $\mathbf{a} \cdot (\mathbf{a} \times \mathbf{b}) = 0$?
- (b) Under what conditions is it true that $(\mathbf{a} \times \mathbf{b}) \cdot (\mathbf{a} \times \mathbf{b}) = 0$?
5. Let A be a square matrix, and suppose that $\det(A^2) = 1$. (“ A^2 ” just means the matrix AA .) What is/are the possible value(s) of $\det A$?
6. Find an example of a 2×2 matrix A other than $\mathbf{0}$ (the zero matrix) with the property that $A^2 = \mathbf{0}$.
7. Which of the following is an actual piece of currency that the U.S. Mint is now issuing? (*Hint*: Your humble instructor has an example thereof in his possession at this very moment.)
- (a) a \$20 bill with a portrait of late-night talk-show host Conan O’Brien
- (b) a nickel on which *E pluribus unum* has been replaced by *Quod erat demonstrandum*
- (c) a \$1 coin featuring Sacagawea
- (d) a quarter honoring “Canada, the 51st state”