18.024–ESG Problem Set 10

Pramod N. Achar

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Tuesday

1. Exercises 1 and 2 in Section 11.15 of Apostol, Volume II.
2. (Optional) Exercises 3–8 in Section 11.15 of Apostol, Volume II. These are just like the integrals in the preceding problems. I don't want to assign an inordinate number of purely computational problems, but it takes some practice to get used to setting up the limits on double integrals, so if you are uncomfortable with the process after the preceding problem, you may wish to try your hand at a few more.

Thursday

3. Exercises 9 and 11 in Section 11.18 of Apostol, Volume II.
4. Let r be a constant. Compute the (four-dimensional) volume of the set
$$B^4(r) = \{\mathbf{x} \in \mathbb{R}^4 \mid \|\mathbf{x}\| \le r\}.$$
5. (Optional—This problem is not conceptually hard, but the computation involved is *very* tedious.) Define $B^n(r)$ as above, but with $\mathbb{R}^4$ replaced by $\mathbb{R}^n$. Thus $B^1(r)$ is the line segment $[-r, r]$; $B^2(r)$ is the disk in $\mathbb{R}^2$ of radius r centered at the origin; and $B^3(r)$ is the solid three-dimensional ball of radius r. The volume (length) of $B^1(r)$ is just $2r$; and as you remember from high school, $B^2(r)$ has volume (area) πr^2, and $B^3(r)$ has volume $\frac{4}{3}\pi r^3$. From the preceding exercise, you know the volume of $B^4(r)$. Compute the volumes of a few more of these, say $B^5(r)$ and $B^6(r)$. Then, conjecture a general formula for the n-dimensional volume of $B^n(r)$. Your answer should depend on n and r. (*Hint*: You should actually find *two* general formulas, one of which works for odd n, and the other for even n.) If you have a great deal of stamina for doing ugly high-dimensional integrals, prove your conjectured formulas by induction.

Friday

6. Exercises 1(a) and 1(d) in Section 11.22 of Apostol, Volume II.
7. Exercise 4 in Section 11.22 of Apostol, Volume II.