18.024–ESG Problem Set 3

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Tuesday

1. Exercises 12 and 15 in Section 16.20 of Apostol, Volume I.

2. Consider the 2×3 matrix

$$P = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}.$$

   (a) Show that P has no left inverse, and find at least two right inverses for it. (Remember that any left or right inverse should be a 3×2 matrix.)

   (b) (Optional) Find the set of all right inverses of P. If you think of 3×2 matrices as a way of writing down vectors in $\mathbb{R}^6$, then the set of all right inverses of P is some plane in $\mathbb{R}^6$. Give a parametric equation for this plane. What is its dimension?

Thursday

3. Compute the determinants of the following matrices:

$$\begin{pmatrix} 2 & 1 & 1 \\ 1 & 4 & -4 \\ 1 & 0 & 2 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} 3 & 0 & 8 \\ 5 & 0 & 7 \\ -1 & 4 & 2 \end{pmatrix}.$$

   (These are from Exercise 1 in Section 3.6 of Apostol, Volume II.)

4. Recall that the *transpose* of a $k \times n$ matrix is the $n \times k$ matrix obtained by flipping the matrix along the diagonal. We will use a superscript "t" to indicate the transpose of a matrix; thus, the transpose of the matrix P given in Problem 2 is

$$P^{\mathrm{t}} = \begin{pmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{pmatrix}.$$

   If a matrix is square, then of course its transpose is also square. Show that if A is a square matrix, then

$$\det A = \det A^{\mathrm{t}}.$$

(*Hint*: First show it to be true for the elementary Gauss-Jordan matrices. When proving it in general, you may use without proof the following fact about matrix multiplication and transposes:

$$(AB)^{\mathrm{t}} = B^{\mathrm{t}} A^{\mathrm{t}}.$$

You should remember this fact from high school!)

5. In class, we gave a formula for computing determinants by expanding along a column. Using the previous exercise, show that determinants can be computed by expanding along a row as well.

Friday

6. Exercises 1(a), 1(b), and 2(a) in Section 13.11 of Apostol, Volume I.