18.024–ESG Problem Set 4

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Thursday

1. Let $\mathbf{f}, \mathbf{g} : A \to \mathbb{R}^3$ be two differentiable functions, where $A \subset \mathbb{R}$. Prove the product rule for cross product; *viz.*,

$$D(\mathbf{f} \times \mathbf{g}) = D\mathbf{f} \times \mathbf{g} + \mathbf{f} \times D\mathbf{g}.$$

   Be careful when manipulating this formula: remember that the cross product, unlike multiplication in $\mathbb{R}$, is not commutative.

2. Exercises 1 and 6 in Section 14.4 of Apostol, Volume I.

3. Exercises 8 and 12 in Section 14.4 of Apostol, Volume I. For Exercise 12, recall that the hyperbolic cosine function is defined by the formula

$$\cosh x = \frac{e^x + e^{-x}}{2}.$$

4. (Optional) Let $\mathbf{f}$ and $\mathbf{g}$ be two *matrix-valued* functions defined on some domain $A \subset \mathbb{R}$. We can define the notions of limit, derivative, and integral for matrix-valued functions entry-by-entry, just as we did for vector-valued functions. Suppose that $\mathbf{f}$ takes values that are $k \times n$ matrices, and $\mathbf{g}$ takes values that are $n \times m$ matrices, so that it makes sense to take the product $\mathbf{f}(t)\mathbf{g}(t)$. Prove that there is a product rule for matrix-valued functions; *viz.*, if $\mathbf{f}$ and $\mathbf{g}$ are differentiable, then

$$D(\mathbf{f}\mathbf{g}) = D\mathbf{f}\,\mathbf{g} + \mathbf{f}\,D\mathbf{g}.$$

   As with the cross product, remember that matrix multiplication is not commutative.

Friday

5. Exercises 1 and 2 in Section 14.7 of Apostol, Volume I.

6. Exercise 7 in Section 14.7 of Apostol, Volume I.