18.024–ESG Problem Set 5

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Tuesday

1. For each of the following curves, find the unit tangent vector $\mathbf{t}$ and the unit normal vector $\mathbf{n}$ as functions of t. Then find a parametric equation for the osculating plane to the curve at $t = 0$.

   (a) $\mathbf{r}(t) = (3t - t^3)\mathbf{i} + 3t^2\mathbf{j} + (3t + t^3)\mathbf{k}$

   (b) $\mathbf{r}(t) = \cos t\,\mathbf{i} + \sin t\,\mathbf{j} + e^t\mathbf{k}$

2. Compute the curvature $\kappa(t)$ for each of the two curves given in Problem 1.

3. (Optional) What is the nonmathematical meaning of the verb *to osculate*?

Friday

4. Exercise 1 in Section 14.19 of Apostol, Volume I.

5. Suppose we are given a curve in $\mathbb{R}^2$ in polar coordinates, $r = f(\theta)$ ($a \leq \theta \leq b$). Show that the arc length of this curve is given by

$$\int_{\theta=a}^{b} \sqrt{f(\theta)^2 + f'(\theta)^2}.$$

   (*Hint*: First describe the curve parametrically with a function of the form $\mathbf{r} : [a, b] \to \mathbb{R}^2$, and then use the formula we proved in class for the arc length of a curve given in this form.)