18.024–ESG Problem Set 6

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Starting this week, we will primarily be using Apostol, Volume II.

Tuesday

1. Show that

$$\lim_{(x,y)\to(0,0)} \frac{y^2}{x^2+y^2}$$

does not exist. (*Hint*: What happens if you fix $x = 0$ and let $y \to 0$? What if you fix $y = 0$ and let $x \to 0$?)

2. Exercises 1, 13, and 16 in Section 8.9 of Apostol, Volume II.

Thursday

3. Exercises 1(a) and 2(a) in Section 8.14 of Apostol, Volume II.

4. Let $\mathbf{g} : \mathbb{R}^3 \to \mathbb{R}^3$ be the function $\mathbf{g}(x, y, z) = (x, y, z)$. Find a function $f : \mathbb{R}^3 \to \mathbb{R}$ such that $\nabla f = \mathbf{g}$.

5. Exercise 11 in Section 8.14 of Apostol, Volume II.

Friday

6. Consider the following three "hypersurfaces" in $\mathbb{R}^4$:

$$\begin{aligned} x^2 + y^2 - z^2 + 3w^2 &= 4 \\ e^{x-y} - zw &= 0 \\ x + y^2z^2w^2 - 2yzw &= 0 \end{aligned}$$

Each of these passes through the point $(1, 1, 1, 1)$, and has a tangent 3-plane at that point. The intersection of these tangent 3-planes is a 1-plane; *i.e.*, a line. In other words, there is exactly one line passing through the point $(1, 1, 1, 1)$ that is tangent to all three of the above hypersurfaces. Find a parametric equation for that line. (*Hint*: Use the gradient to find a normal vector to each hypersurface. The line will have to lie along a direction perpendicular to all three normal vectors. To find this direction, it may be helpful to recall the cross-product analogue for $\mathbb{R}^4$ as described in class by Gerry: the vector in $\mathbb{R}^4$ given by the mnemonic formula

$$\det\begin{pmatrix} \mathbf{e}_1 & \mathbf{e}_2 & \mathbf{e}_3 & \mathbf{e}_4 \\ \longleftarrow & & \mathbf{a} & \longrightarrow \\ \longleftarrow & & \mathbf{b} & \longrightarrow \\ \longleftarrow & & \mathbf{c} & \longrightarrow \end{pmatrix}$$

is perpendicular to each of the vectors **a**, **b**, and **c**.)

7. Exercise 2 in Section 9.13 of Apostol, Volume II.