18.024–ESG Problem Set 9

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Thursday

1. Let $S = \mathbb{R}^n \setminus \{\mathbf{0}\}$, and let $p \in \mathbb{R}$ be some real constant. Define a vector field $\mathbf{f} : S \to \mathbb{R}^n$ by

$$\mathbf{f}(\mathbf{x}) = \|\mathbf{x}\|^p \mathbf{x}.$$

   It turns out that this vector field is always a gradient.

   (a) Find a potential function for it. (It may be helpful to sketch this vector field in the one-dimensional case, and finding the answer in that case may give you an idea of what to expect in the n-dimensional case. Note: your answer should treat the case $p = -2$ separately.)

   (b) For what values of p can you extend $\mathbf{f}$ to be defined at $\mathbf{0}$ as well, such that the new vector field is continuous at $\mathbf{0}$?

Friday

2. Exercises 1, 2, and 3 in Section 11.9 of Apostol, Volume II.

3. Let $Q = [-1, 1] \times [-1, 1]$, and define a function $f : Q \to \mathbb{R}$ by

$$f(x, y) = \begin{cases} 1 & \text{if } x^2 + y^2 \leq 1, \\ 0 & \text{otherwise.} \end{cases}$$

   Sketch a graph of this function, and compute $\iint_Q f$. What is a geometric interpretation of the number you just computed?