18.024–ESG Course Description

Pramod N. Achar

Spring 2000

Instructor:	Pramod N. Achar
Office:	Rm. 2-251
Phone:	x3-7566 (office); 776-9956 (home)
E-mail:	`pramod@math.mit.edu`
Class Meetings:	TRF3 in Rm. 24-621
Text:	T. M. Apostol, *Calculus*, 2d ed., vols. I and II.
Grading:	40% problem sets; 40% exams; 20% participation

Overview. The first semester of calculus was about functions whose domain and codomain were $\mathbb{R}$, or subsets thereof. This semester, we will study functions whose domain and codomain are subsets of $\mathbb{R}^n$. Three major questions we want to try to answer this semester are:

- What does it mean for a function in this setting to be differentiable? How do we compute its derivative(s)?
- What does it mean for such a function to be integrable? How do we compute integrals in this setting?
- What is the relationship between differentiation and integration? Is there a higher-dimensional analogue of the Fundamental Theorem of Calculus?

We will treat differentiation thoroughly: it is not conceptually different in $\mathbb{R}^{100}$ from differentiation in $\mathbb{R}^1$; the main new thing to learn is the extra algebraic bookkeeping to keep track of all the extra variables. Integration, on the other hand, is a subtler notion. We will talk about several kinds of integration in multiple variables, and we will obtain a few analogues of the Fundamental Theorem in dimensions up to 3 (these are Green's, Stokes', and Gauss' Theorems).

Many multivariable calculus students find these theorems to be the most confusing topic of the semester. They are significantly more complicated to state than the Fundamental Theorem, and the ideas of "curl," "grad," and "div" are not nearly as intuitive as the idea of "derivative." The problem is that vectors and matrices are not really the proper language in which to state them: one needs "tensors" and "differential forms," the subject matter of 18.101. In my own experience, Stokes' Theorem *et al.* made much more sense in the context of the more general integration theorems of 18.101, which apply in any number of dimensions.

We will not talk about tensors or differential forms in this class, but I will try to provide some of the higher-dimensional context that was helpful to me. In particular, I hope to have part or all of the final problem set be, "State and prove an analogue of the Fundamental Theorem in 4 dimensions." (Don't worry; I will give additional hints.) This is something that 18.02 students never see, but my hope is that seeing this higher-dimensional theorem will give you some feel for what the pattern is, and what the 3- and lower-dimensional theorems that are so ubiquitous in physics are all about.

Outline. The course will be divided into four units, each occupying approximately one-quarter of the semester, as follows:

1. *Vectors and linear algebra.* Here we lay down the algebraic foundations for working in higher dimensions: vectors, matrices, and linear subspaces. This unit is the analogue of our study of arithmetic in $\mathbb{R}$ using the field axioms.

2. *Derivatives.* The first half of this unit will focus on functions $f : \mathbb{R} \to \mathbb{R}^n$, and the second half will work in the more general setting of functions $f : \mathbb{R}^n \to \mathbb{R}^m$. Many of our theorems about derivatives from last semester will reappear, albeit in slightly modified form.

3. *Integration of scalar-valued functions.* In this unit, we look at the easiest generalizations of integration to higher dimensions: we look at integrals of functions $\mathbb{R}^n \to \mathbb{R}$ over curves and regions. But something seems to be missing: there is a good analogue of the Fundamental Theorem only for integrals along curves.

4. *Integrals of vector-valued functions; Stokes' Theorem.* This unit involves the biggest conceptual leaps of the semester, and culminates with our *pièce de résistance*. One basic problem is that of definition: what does it mean to integrate a vector-valued function over a 2- or 3-dimensional region and get a scalar answer at the end? But the reward for tackling this will be Green's, Stokes', and Gauss' Theorems, which tell us how integrals over regions of different dimensions relate to one another, and give us a broader context for understanding the Fundamental Theorem.

Grading. There will be approximately twelve weekly problem sets, each graded out of 20 points; and there will be a one-hour exam after each of the four units, each graded out of 100 points. The problem sets and exams each account for 40% of the final grade. For the remaining 20% of the grade, I would like to have each of you present proofs in class from time to time.