

Extra Credit

Due: October 25, 2005

Suppose $f(x)$ and $g(x)$ are two functions whose derivatives you know. There are various ways to combine them to get a new function. In class, we have developed various rules that tell us how to find the derivative of the combined function in terms of $f'(x)$ and $g'(x)$. For example:

- If you add them, the Sum Rule says

$$\frac{d}{dx}(f(x) + g(x)) = f'(x) + g'(x).$$

- If you multiply them, the Product Rule tells you that

$$\frac{d}{dx}(f(x)g(x)) = f(x)g'(x) + g(x)f'(x).$$

- If you divide them, the Quotient Rule tells you that

$$\frac{d}{dx} \left(\frac{f(x)}{g(x)} \right) = \frac{g(x)f'(x) - f(x)g'(x)}{g(x)^2}.$$

- If you compose them, the Chain Rule tells you that

$$\frac{d}{dx}(f(g(x))) = f'(g(x))g'(x).$$

Extra Credit Problem. Find an “Exponentiation Rule” that tells you what to do when you raise one function to the power of another. In other words, find a formula for

$$\frac{d}{dx} \left(f(x)^{g(x)} \right)$$

in terms of $f'(x)$ and $g'(x)$. You must also prove that your formula is correct. I will give no credit if you just write down the formula without any reasoning.

If you do this correctly, I will add 5 points to the total of your exam scores at the end of the semester.