Problem Set 4
Due: November 3, 2005

1. (Geck-Pfeiffer, Exercise 7.1) Let \( k \) be a field, and let \( M_n(k) \) be the algebra of \( n \times n \) matrices over \( k \). Show that the only symmetrizing traces on \( M_n(k) \) are scalar multiples of the ordinary trace function.

2. (Geck-Pfeiffer, Exercise 7.6) Suppose \( H \) is a symmetric algebra over \( k \), and assume the symmetrizing trace is a linear combination of characters of simple modules for \( H \). Show that \( H \) is semisimple.

3. In class, I mentioned the following algebra and claimed that it was not semisimple:

\[
H = \mathbb{C}[x]/(x^2), \quad \tau : H \to \mathbb{C} \text{ defined by } \tau(1) = \tau(x) = 1.
\]

Find a simple module for this algebra whose Schur element is 0.

4. Let \( \mathcal{H} \) be the generic Hecke algebra for a finite Coxeter group \( W \), and let \( V \) be a simple module for \( \mathcal{H} \). The generic degree of \( V \), denoted \( d_V \), is defined to be the following quotient of Schur elements:

\[
d_V = \frac{\text{ind}}{\text{cv}}.
\]

Let \( \theta_1 : A \to \mathbb{C} \) be the usual specialization sending all indeterminates to 1, so that \( \mathcal{H}_{\theta_1} \simeq \mathbb{C}[W] \). What can you say about \( \theta_1(d_V) \)? (Hint: It may be helpful to know the following fact about characters of finite groups: if \( V \) is an irreducible representation of the finite group \( G \), then

\[
\sum_{g \in G} \chi_V(g)\chi_V(g^{-1}) = |G|.
\]

Here, \( \chi_V(g) \) is, of course, the trace of the action of \( g \) on \( V \).)