Problem Set 4

Due: November 3, 2005

1. (Geck-Pfeiffer, Exercise 7.1) Let k be a field, and let $M_n(k)$ be the algebra of $n \times n$ matrices over k. Show that the only symmetrizing traces on $M_n(k)$ are scalar multiples of the ordinary trace function.

2. (Geck-Pfeiffer, Exercise 7.6) Suppose H is a symmetric algebra over k, and assume the symmetrizing trace is a linear combination of characters of simple modules for H. Show that H is semisimple.

3. In class, I mentioned the following algebra and claimed that it was not semisimple:

$$H = \mathbb{C}[x]/(x^2), \qquad \tau : H \to \mathbb{C} \text{ defined by } \tau(1) = \tau(x) = 1.$$

   Find a simple module for this algebra whose Schur element is 0.

4. Let $\mathcal{H}$ be the generic Hecke algebra for a finite Coxeter group W, and let V be a simple module for $\mathcal{H}$. The *generic degree* of V, denoted d_V, is defined to be the following quotient of Schur elements:

$$d_V = c_{\text{ind}}/c_V.$$

   Let $\theta_1 : A \to \mathbb{C}$ be the usual specialization sending all indeterminates to 1, so that $\mathcal{H}_{\theta_1} \simeq \mathbb{C}[W]$. What can you say about $\theta_1(d_V)$? (*Hint:* It may be helpful to know the following fact about characters of finite groups: if V is an irreducible representation of the finite group G, then

$$\sum_{g \in G} \chi_V(g)\chi_V(g^{-1}) = |G|.$$

   Here, $\chi_V(g)$ is, of course, the trace of the action of g on V.)