1. Let \( p : Y \to X \) be a finite-to-one covering map. If \( \mathcal{E} \) is a local system on \( Y \), show that \( p_* \mathcal{E} \) is a local system on \( X \). If \( p \) is not finite-to-one, the proof of the previous statement does not go through, but it can be fixed up by imposing an additional condition on \( \mathcal{E} \). Find such a condition, and prove that with this extra condition, \( p_* \mathcal{E} \) is again a local system. (Hint: If \( Y \) is compact, no additional condition on \( \mathcal{E} \) is needed.)

2. Recall that there is a one-to-one correspondence between covering spaces over \( X \) (up to isomorphism) and subgroups of its fundamental group \( \pi_1(X,x_0) \) (up to conjugacy. Suppose \( p : Y \to X \) is a covering map corresponding to the subgroup \( H \subset \pi_1(X,x_0) \). Let \( E = \mathbb{C}[\pi_1(X,x_0)/H] \), the vector space of formal linear combinations of cosets of \( H \) with complex coefficients. There is an obvious representation of \( \pi_1(X,x_0) \) on \( E \). Show that the local system \( p_* \mathcal{E} \) is the one corresponding to this representation.

3. Show that the category of sheaves of abelian groups on a fixed topological space \( X \) is an abelian category.

4. Let \( S : \mathcal{A} \to \mathcal{B} \) and \( T : \mathcal{B} \to \mathcal{A} \) be an adjoint pair of functors. Show that \( S \) is right-exact and that \( T \) is left-exact. (Be sure to do this using the language of categories—you cannot talk about “elements” of kernels and images, because in a general abelian category, the concept of “element” may not make sense.) It immediately follows that the functors

\[
\text{Hom}(G, \cdot), \quad \text{Hom}(\cdot, G), \quad f_*
\]

are left-exact (here \( G \) is a sheaf and \( f : X \to Y \) is a continuous map), and

\[
\cdot \otimes G, \quad f^{-1}, \quad j_!
\]

are right-exact (here \( j : U \to X \) is an open inclusion). Finally, note that \( j^{-1} \) (again, \( j \) is an open inclusion) is exact, because it has adjoints on both sides: \( j_* \) is its right adjoint, and \( j_! \) is its left adjoint.

5. Show that \( f^{-1} \) is actually an exact functor for any continuous map \( f : X \to Y \), not just an open inclusion.

6. Show that if \( j : U \to X \) is an open inclusion, then \( j_! \) is an exact functor. (It suffices to show that it is right-exact.)