
2. *Ibid.*, Exercise II.5.5 (page 120). (*Note:* See Exercise II.5.4 for the definition of “f(y).”)

3. Let $A^\bullet \xrightarrow{f} B^\bullet \xrightarrow{g} C^\bullet \xrightarrow{} A[1]^\bullet$ be a distinguished triangle (in $K(A)$ or $D(A)$). Show that $g \circ f = 0$.

4. Let $0 \xrightarrow{} A^\bullet \xrightarrow{f} B^\bullet \xrightarrow{g} C^\bullet \xrightarrow{} 0$ be a short exact sequence in $C(A)$. Define a morphism $\theta : \text{cone}^\bullet f \rightarrow C^\bullet$ by the matrix $(0 \ g)$. Show that $\theta$ is indeed a morphism of complexes (i.e., that it commutes with differentials) and that the following diagram commutes:

![Diagram](A diagram showing the commutative property of $\theta$)

Finally, show that $\theta$ is a quasi-isomorphism. Deduce that in $D(A)$ (but not necessarily in $K(A)$), there is a distinguished triangle of the form $A^\bullet \xrightarrow{f} B^\bullet \xrightarrow{g} C^\bullet \xrightarrow{} A[1]^\bullet$.

5. Prove that the octahedral property holds for distinguished triangles in $D(A)$.

6. Given a complex $A^\bullet$, define new complexes $\tau_{\leq 0} A^\bullet$ and $\tau_{\geq 1} A^\bullet$ by

$$ (\tau_{\leq 0} A)^i = \begin{cases} A^i & \text{if } i < 0, \\ \ker d_A^0 & \text{if } i = 0, \\ 0 & \text{if } i > 0 \end{cases} \quad \text{and} \quad (\tau_{\geq 1} A)^i = \begin{cases} 0 & \text{if } i < 1, \\ \text{cok} d_A^0 & \text{if } i = 1, \\ A^i & \text{if } i > 1. \end{cases} $$

There is an obvious way to define differentials (so that $\tau_{\leq 0} A^\bullet$ and $\tau_{\geq 1} A^\bullet$ are actually complexes), as well as obvious morphisms $\tau_{\leq 0} A^\bullet \rightarrow A^\bullet$ and $A^\bullet \rightarrow \tau_{\geq 1} A^\bullet$. Find all of these, and then show that there is a distinguished triangle

$$ \tau_{\leq 0} A^\bullet \rightarrow A^\bullet \rightarrow \tau_{\geq 1} A^\bullet \rightarrow \tau_{\leq 0} A[1]^\bullet. $$

What can you say about the cohomology objects of the three complexes $\tau_{\leq 0} A^\bullet$, $A^\bullet$, and $\tau_{\geq 1} A^\bullet$?