Problem Set 1

Due: September 8, 2008

1. Humphreys, p. 15, Exercise 1.
2. Humphreys, p. 15, Exercise 5. This question is slightly ill-posed; a better formulation is this: find an open subset $U \subset \mathbb{A}^2$ that, together with its ring of regular functions $K[U]$, is not an "abstract affine variety" as defined in class.
3. Consider the following algebraic group:

$$G = \{(x,y) \in \mathbb{A}^2 \mid x^2 + y^2 = 1\} \qquad \text{with} \qquad (x,y)(u,v) = (xu - yv, xv + yu).$$

   Show that $G \simeq GL_1(K)$. *Note:* You must find a map between the two that is both an isomorphism of affine varieties and a group isomorphism. You will need to use the fact that K is algebraically closed: over $\mathbb{R}$, for instance, the group defined above is the circle, which is certainly not isomorphic to $GL_1(\mathbb{R})$.
4. Humphreys, p.15, Exercise 6.
5. Humphreys, p.16, Exercise 10.