Problem Set 1
Due: September 8, 2008

1. Humphreys, p. 15, Exercise 1.

2. Humphreys, p. 15, Exercise 5. This question is slightly ill-posed; a better formulation is this: find an open subset $U \subset \mathbb{A}^2$ that, together with its ring of regular functions $K[U]$, is not an “abstract affine variety” as defined in class.

3. Consider the following algebraic group:

   \[ G = \{(x, y) \in \mathbb{A}^2 \mid x^2 + y^2 = 1\} \quad \text{with} \quad (x, y)(u, v) = (xu - yv, xv + yu). \]

   Show that $G \simeq GL_1(K)$. Note: You must find a map between the two that is both an isomorphism of affine varieties and a group isomorphism. You will need to use the fact that $K$ is algebraically closed: over $\mathbb{R}$, for instance, the group defined above is the circle, which is certainly not isomorphic to $GL_1(\mathbb{R})$.
