Problem Set 2

Due: September 24, 2009

1. Auslander–Reiten–Smalø, Chapter 2, Exercise 1.

2. Auslander–Reiten–Smalø, Chapter 2, Exercise 2.

3. The previous exercise and Proposition 2.5 in the textbook both involve equivalences of categories arising from projective modules. Formulate and prove an analogous statement involving injective modules instead. (*Hint*: Rather than proving the equivalence directly, use a duality functor to deduce it from a statement about projectives.)

4. Recall that we defined $\mathrm{Ext}^1(B, A)$ in terms of the projective cover of B. Here is an alternate characterization. Let I be the injective envelope of A, and let Q be the cokernel of the injective map $A \to I$, so that we have a short exact sequence $0 \to A \to I \to Q \to 0$. Consider the induced map

$$\delta : \mathrm{Hom}(B, I) \to \mathrm{Hom}(B, Q).$$

Prove that $\mathrm{Ext}^1(B, A) \cong \mathrm{cok}\, \delta$.