

Problem Set 3

Due: October 29, 2009

1. Auslander–Reiten–Smalø, Chapter 3, Exercise 2.
2. Auslander–Reiten–Smalø, Chapter 3, Exercise 6.
3. Auslander–Reiten–Smalø, Chapter 3, Exercise 7.

The exercises in Chapter 3 have lots of other specific examples of quivers with various properties. It might be interesting to look through the problems even if you don't end up working on any of them.

4. Auslander–Reiten–Smalø, Chapter 4, Exercise 3.
5. Let (Γ, ρ) denote the following quiver with relations:



- (a) Determine the isomorphism classes of indecomposable representations of (Γ, ρ) . (There are five: 2 simple objects; 2 indecomposable projectives that happen to also be injective; and one other that is neither projective nor injective.)
- (b) (Optional—Mandatory if you know about algebraic groups and perverse sheaves) Let $G = SL_2(\mathbb{C})$, and let $B \subset G$ be a Borel subgroup. Let \mathcal{A} denote the category of perverse sheaves on G/B (which is isomorphic to \mathbb{P}^1), stratified by B -orbits. Prove that there is an equivalence of categories between \mathcal{A} and $\text{Rep}(\Gamma, \rho)$. (There isn't any obvious functor between these categories; instead, I think you just have to classify the indecomposables in \mathcal{A} , and then check that the Hom-groups are the same as for the corresponding objects in $\text{Rep}(\Gamma, \rho)$.)