Problem Set 1

Due: January 22, 2014

1. (Do not hand in) Weibel, Exercises 1.1.1, 1.1.2, 1.1.5.
2. (Do not hand in) Show that any isomorphism of chain complexes is a quasi-isomorphism.
3. Weibel, Exercise 1.1.3. Also, give an example of a quasi-isomorphism that is not an isomorphism.
4. Weibel, Exercise 1.1.4.
5. Let $R = \mathbb{C}[t]/(t^2)$. Regarding R as a module over itself, consider the R-module homomorphism $f : R \to R$ given by $f(r) = tr$. Next, let N be the R-module $R/(t) \cong \mathbb{C}[t]/(t)$. Consider the following two chain complexes (with nonzero terms only in degrees 0 and -1):

$$C = (\cdots \to 0 \to R \xrightarrow{f} R \to 0 \to \cdots),$$
$$D = (\cdots \to 0 \to N \xrightarrow{0} N \to 0 \to \cdots).$$

Prove that $H_n(C) \cong H_n(D)$ for all n, but that there is *no* quasi-isomorphism $u : C \to D$ or $u : D \to C$.