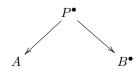
## Problem Set 3

Due: November 3, 2015

1. Let  $\mathcal{A}$  be an abelian category with enough projectives, and let  $A \in \mathcal{A}$ . Regard A as a chain complex concentrated in degree 0. Choose a projective resolution  $P^{\bullet} \to A$ . (Recall that with our new conventions,  $P^{\bullet}$  looks like  $\cdots \to P^{-2} \to P^{-1} \to P^0 \to 0$ .) Show that for any object  $B^{\bullet}$  in  $D(\mathcal{A})$  and any morphism  $f: A \to B^{\bullet}$ , f can be represented by a roof of the form



2. Now, assume  $B \in \mathcal{A}$ . Using the previous problem, show that

$$\operatorname{Hom}_{D(\mathcal{A})}(A, B[n]) \cong \begin{cases} 0 & \text{if } n < 0, \\ \operatorname{Ext}^{n}(A, B) & \text{if } n \ge 0. \end{cases}$$

*Remark.* Here is a generalization of the "n < 0" case above. (This isn't a homework problem; it relies on facts about  $D(\mathcal{A})$  that we haven't gotten to yet.) Let  $D(\mathcal{A})^{\leq n}$  denote the subcategory of  $D(\mathcal{A})$ consisting of objects  $X^{\bullet}$  such that  $H^{i}(X^{\bullet}) = 0$  for i > n. Similarly, let  $D(\mathcal{A})^{\geq n}$  be the subcategory consisting of  $X^{\bullet}$  such that  $H^{i}(X^{\bullet}) = 0$  for i < n. If  $X^{\bullet} \in D(\mathcal{A})^{\leq n}$  and  $Y^{\bullet} \in D(\mathcal{A})^{\geq n+1}$ , then

$$\operatorname{Hom}_{D(\mathcal{A})}(X^{\bullet}, Y^{\bullet}) = 0.$$

Convince yourself that the "n < 0" case of the problem is indeed a special case of this fact.

3. Use the fact that  $\operatorname{Ext}^{1}(A, B) \cong \operatorname{Hom}_{D(\mathcal{A})}(A, B[1])$  to give a new proof of the fact that

 $\operatorname{Ext}^{1}(A, B) \cong \{ \text{equivalence classes of short exact sequences } 0 \to B \to X \to A \to 0 \}.$