

**Problem Set 1**  
*Due: January 26, 2015*

1. (Not to hand in) Let  $Q$  be a quiver, and let  $k$  be a field. Show the category  $\text{Rep}_k(Q)$  is equivalent to the category of finite-dimensional  $kQ$ -modules.
2. Let  $Q$  be a quiver with no oriented cycles. (Thanks Jacob!) Classify the simple objects in  $\text{Rep}_k(Q)$ . Is there a Jordan–Hölder theorem for  $\text{Rep}_k(Q)$ ?
3. Let  $Q$  be a quiver with no oriented cycles. Consider the following condition on an object  $(V, \phi)$  of  $\text{Rep}_k(Q)$ :

For every vertex  $s$ , the map  $\bigoplus_{t \rightarrow s} V_t \xrightarrow{\bigoplus \phi_{t \rightarrow s}} V_s$  is injective. (\*)

Prove that an object of  $\text{Rep}_k(Q)$  is projective if and only if it satisfies (\*). As a corollary, deduce that any subobject of a projective object is projective.

4. Let  $Q$  be a quiver with no oriented cycles. Prove that  $\text{Rep}_k(Q)$  has enough projectives. Choose your favorite quiver (not too small!) and determine all the indecomposable projectives in  $\text{Rep}_k(Q)$ .