Problem Set 1
Due: January 26, 2015

1. (Not to hand in) Let $Q$ be a quiver, and let $k$ be a field. Show the category $\text{Rep}_k(Q)$ is equivalent to the category of finite-dimensional $kQ$-modules.

2. Let $Q$ be a quiver with no oriented cycles. (Thanks Jacob!) Classify the simple objects in $\text{Rep}_k(Q)$. Is there a Jordan–H"older theorem for $\text{Rep}_k(Q)$?

3. Let $Q$ be a quiver with no oriented cycles. Consider the following condition on an object $(V, \phi)$ of $\text{Rep}_k(Q)$:

   For every vertex $s$, the map $\bigoplus_{t \to s} V_t \xrightarrow{\oplus \phi_{t \to s}} V_s$ is injective. \hfill (*)

   Prove that an object of $\text{Rep}_k(Q)$ is projective if and only if it satisfies (*). As a corollary, deduce that any subobject of a projective object is projective.

4. Let $Q$ be a quiver with no oriented cycles. Prove that $\text{Rep}_k(Q)$ has enough projectives. Choose your favorite quiver (not too small!) and determine all the indecomposable projectives in $\text{Rep}_k(Q)$. 