18.014–ESG Exam 1

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1. State the completeness axiom for $\mathbb{R}$.

2. Which of the following sets are fields? Which are ordered fields?

	Field?	*Ordered field?*
$\mathbb{Z}$		
$\mathbb{Q}$		
$\mathbb{C}$		

3. State the Archimedean property of $\mathbb{R}$.

4. What is a step function?

5. Let $\lfloor x \rfloor$ denote the greatest integer less than or equal to x. Compute the following integrals:

    (a) $\displaystyle\int_{x=0}^{2} \lfloor x \rfloor$

    (b) $\displaystyle\int_{x=0}^{2} \lfloor 2x \rfloor$

6. What way of forming new partitions from old partitions did we use to show that a sum or product of step functions is again a step function? (*Hint*: Two words.)

7. What is a bounded function?

8. Compute the following integrals:

   (a) $\int_{x=0}^{2} x^2$

   (b) $\int_{x=0}^{2} (5x^3 + 4x)$

9. Let $f : [a, c] \to \mathbb{R}$ be a bounded function. Suppose that $\underline{\int_a^c} f = 2$ and $\overline{\int_a^c} f = 5$. Is f integrable on $[a, c]$?

   Now suppose that b is some number such that $a < b < c$, and suppose that $\int_a^b f = -1$ and $\overline{\int_a^b} f = 2$. Is f integrable on $[a, b]$? What about on $[b, c]$? If f is integrable on either of these intervals, give the value of its integral on that interval.

10. (a) State the triangle inequality.

    (b) Prove that $|x - y| \geq |x| - |y|$.

    (c) Prove that $\big||x| - |y|\big| \leq |x - y|$.