18.014–ESG Problem Set 10

Pramod N. Achar

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Wednesday

1. Compute the first few Taylor polynomials of $\arctan x$ about 0. You should find that all the even-degree coefficients are zero.

2. Observe that $\pi/4 = \arctan 1$; in other words, $\pi = 4 \arctan 1$. Use the highest-degree Taylor polynomial you found in the preceding question to get an estimate of the value of π; then, use Taylor's Theorem to get a bound on the error in your estimate.

   Of course, unless you are much more arithmetically talented than I, you will use a calculator to help evaluate the Taylor polynomial. But back in the old days, before electronic computing equipment was omnipresent, people actually used Taylor polynomials to compute the value of π. Using the above method, you would need a Taylor polynomial of huge degree in order to get an estimate that's correct to just a couple decimal places. (By my quick and possibly erroneous computations on a calculator, the 51^{st}-degree Taylor polynomial gives a value of π correct to one decimal place.) But in 1704, John Machin observed that

$$\pi = 16 \arctan \frac{1}{5} - 4 \arctan \frac{1}{239}.$$

   (This is easily proved using basic trigonometric identities.) The right-hand side of this equation can also be estimated by plugging in $\frac{1}{5}$ and $\frac{1}{239}$ into the Taylor polynomials of $\arctan x$. Using the 11^{th}-degree Taylor polynomial with this formula yields an estimate of π correct to seven decimal places—quite an improvement! (All this information comes from Exercise 10 in Section 7.8 of Apostol. If you are interested by the prospect of computing digits of π by hand, you may work through that problem, provided that you do not hand it in.)

Friday

3. Exercises 1, 3, and 13 in Section 7.17 of Apostol.

4. Exercises 28 and 29 in Section 7.17 of Apostol.