

# 18.014–ESG Problem Set 3

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*Monday*

1. Read Exercise 1 in Section 1.15 of Apostol. Choose any three of the integrals and compute them. Sketch graphs of the functions you chose.

For the next three problems, it may be helpful to read over the proofs of Theorems 1.3 and 1.8 given in Section 1.15 of Apostol.

2. Exercise 13 in Section 1.15 of Apostol.
3. Exercise 14 in Section 1.15 of Apostol.
4. Exercise 15 in Section 1.15 of Apostol.

*Wednesday*

5. Recall that

$$\int_a^b f = \inf \left\{ \int_a^b s \mid s \text{ is a step function and } s(x) \leq f(x) \text{ for } x \in [a, b] \right\}.$$

Of course, in order to take infimum on the right-hand side, we need to know that the set is nonempty and bounded below. Show that if  $f$  is a bounded function on  $[a, b]$ , then in fact that set is nonempty and bounded below.

*Friday*

6. Exercise 13 in Section I 4.10 of Apostol. This problem has three parts, the last of which is to prove the following inequalities:

$$\sum_{k=1}^{n-1} k^p < \frac{n^{p+1}}{p+1} < \sum_{k=1}^n k^p.$$

7. Choose three of the integrals among the first twenty exercises in Section 1.26 of Apostol, and compute them.