18.014–ESG Problem Set 7

Pramod N. Achar

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Monday

1. Exercise 9 in Section 4.9 of Apostol.
2. Exercise 16 in Section 4.9 of Apostol.

Wednesday

3. In class, we proved that $D(x^n) = nx^{n-1}$ for $n \in \mathbb{Z}^+$. Using this fact, the chain rule, and the rule for differentiating inverse functions, prove the formulas given below. Assume that all functions have domain $\mathbb{R} \setminus \{0\}$.
   (a) $D(x^n) = nx^{n-1}$ for all $n \in \mathbb{Z}$ (in other words, you have to prove it when n is zero or negative).
   (b) $D(\sqrt[n]{x}) = \frac{1}{n}(\sqrt[n]{x})^{1-n}$ for all $n \in \mathbb{Z} \setminus \{0\}$.
   (c) $D(x^r) = rx^{r-1}$ where $r \in \mathbb{Q}$.
4. Exercises 15 and 16 in Section 4.12 of Apostol.
5. Exercise 18 in Section 4.12 of Apostol.

Friday

6. Exercise 2 in Section 4.15 of Apostol.
7. Suppose that $f : [a, c] \to \mathbb{R}$ is continuous, and that its first and second derivatives exist on (a, c). Furthermore, suppose that the linear function $g : [a, c] \to \mathbb{R}$ given by $g(x) = mx + k$ (where m and k are constants) has the property that

   $$g(a) = f(a), \qquad g(b) = f(b), \qquad g(c) = f(c),$$

   where b is some point such that $a < b < c$. Show that there is some point $x_0 \in (a, c)$ such that $f''(x_0) = 0$. (*Hint*: What can you say about the difference quotients

   $$\frac{f(b) - f(a)}{b - a} \qquad \text{and} \qquad \frac{f(c) - f(b)}{c - b}?$$

   You will need to apply the Mean-Value Theorem three times to solve this problem.)