18.014–ESG Problem Set 9

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Monday

1. (a) Show that if n is a positive integer, then

$$\left(1+\frac{1}{n}\right)^n \leq e \leq \left(1+\frac{1}{n}\right)^{n+1}.$$

(*Hint*: Show by computing $\int_0^{1/n} \exp$ that

$$\frac{1}{n} \leq \sqrt[n]{e} - 1 \leq \frac{\sqrt[n]{e}}{n}.$$

Use this inequality to derive the above inequality.)

(b) Plug in some large value of n into the above inequality to get an estimate on the value of e.

2. Let $h : \mathbb{R}^+ \to \mathbb{R}^+$ be defined by $h(x) = x^x$. Compute Dh. (*Hint*: Rewrite $h(x)$ as $e^{x \ln x}$.)

3. Let $f : A \to \mathbb{R}^+$ and $g : A \to R$ be continuous, differentiable functions. Since f takes only positive values, $f(x)^{g(x)}$ is always defined. Derive an "Exponential Rule" for the derivative of f^g in terms of the derivatives of f and g.

Wednesday

4. Exercises 19, 25, 30, and 38 from Section 6.22 of Apostol, and Exercises 2 and 5 from Section 6.25 of Apostol.

Friday

5. Compute the first few Taylor polynomials (about 0) of the function $f(x) = 1/(1+x)$. Guess a general formula for its nth-order Taylor polynomials.

6. Using Taylor polynomials, compute

$$\lim_{x \to 0} \frac{x^3 \cos x}{\sin x - x}.$$