18.03–ESG Exam 1

Pramod N. Achar

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Find the general solutions to the following differential equations. Where initial conditions are given, find the particular solution as well.

1. $xy' - 3y = x^3; \qquad y(1) = 10$

2. $(x^2 + y^2 - y)\,dx + x\,dy = 0$ (Use grouping.)

3. $(4x + 3y^3)\,dx + 3xy^2\,dy = 0$ (Use an integrating factor of the form x^m.)

4. $\dfrac{d^3y}{dx^3} + 10\dfrac{d^2y}{dx^2} + 25\dfrac{dy}{dx} = 0$

5. $4y'' + 16y' + 25y = 0$

6. Give a substitution that would enable you to solve each of the following equations, but do not actually solve them. In cases where the substitution has a special name, give the name as well.

   (a) $\dfrac{dy}{dx} = (x + y + 1)^{1/2}$

   (b) $(x - y)y' = x + y$

   (c) $x^2y' + 2xy = 5y^4$

7. What does it mean for the equation $M(x, y)\,dx + N(x, y)\,dy = 0$ to be exact?

8. $y_1 = x^2$ and $y_2 = x^3$ are both solutions of the inital-value problem

$$x^2\frac{d^2y}{dx^2} - 4x\frac{dy}{dx} + 6y = 0; \qquad y(0) = 0, \quad y'(0) = 0.$$

   Why does this not contradict the Existence and Uniqueness Theorem for second-order linear equations?

9. (Optional) *Homogenous* and *homogeneous* are both English words. Which of these applies to differential equations? Which applies to milk?