18.03–ESG Exam 3

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Answer three of the four questions in Problems 1 and 2.

1. Compute the Laplace transforms of the following functions.

   (a) $f(t) = \begin{cases} 1 - t & \text{if } t \leq 1 \\ 0 & \text{if } t \geq 1 \end{cases}$

   (b) $f(t) = te^{2t} \cos 3t$

2. Solve the following initial-value problems using Laplace transforms.

   (a) $x'' + 4x' + 4x = 1 + \delta(t - 2);$ $x(0) = 0,\ x'(0) = 0$

(b) $x'' + 4x' + 8x = e^{-t}$; $x(0) = 0, x'(0) = 0$

3. Evaluate $\mathcal{L}^{-1}\left\{\dfrac{1}{s^2(s-a)}\right\}$ using convolution.

4. Solve the following system by any method.

$$x' = -3x + 2y$$
$$y' = -3x + 4y$$

5. Which of the following is denoted by the symbol $\delta(t)$?

    (a) a number

    (b) a function

    (c) a distribution, or generalized function

    (d) the artist formerly known as Prince