18.03–ESG Exam 4

Pramod N. Achar

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1. Find the general solution to the following nonhomogeneous linear system. You may use either undetermined coefficients or variation of parameters.

$$\mathbf{x}' = \begin{bmatrix} 4 & -1 \\ 5 & -2 \end{bmatrix} \mathbf{x} + \begin{bmatrix} e^{-t} \\ 2e^{-t} \end{bmatrix}$$

2. Compute the matrix exponential $e^{\mathbf{A}t}$, where $\mathbf{A} = \begin{bmatrix} 4 & -1 \\ 5 & -2 \end{bmatrix}$.

3. Find the critical points of the following nonlinear system, and determine the type and stability of each. Then, draw a sketch showing some typical solutions to the system. (*Hint*: There should be four critical points.)

$$x' = x^2 - 2x - xy$$
$$y' = y^2 - 4y + xy$$

4. Enjoy the winter break!