CHARACTERISTIC VARIETIES OF ARRANGEMENTS

DANIEL C. COHEN$^1$ AND ALEXANDER I. SUCIU$^2$

Abstract. The $k$th Fitting ideal of the Alexander invariant $B$ of an arrangement $A$ of $n$ complex hyperplanes defines a characteristic subvariety, $V_k(A)$, of the algebraic torus $(\mathbb{C}^n)^n$. In the combinatorially determined case where $B$ decomposes as a direct sum of local Alexander invariants, we obtain a complete description of $V_k(A)$. For any arrangement $A$, we show that the tangent cone at the identity of this variety coincides with $R^1_k(A)$, one of the cohomology support loci of the Orlik-Solomon algebra. Using work of Arapura, we conclude that all irreducible components of $V_k(A)$ which pass through the identity element of $(\mathbb{C}^n)^n$ are combinatorially determined, and that $R^1_k(A)$ is the union of a subspace arrangement in $\mathbb{C}^n$, thereby resolving a conjecture of Falk. We use these results to study the reflection arrangements associated to monomial groups.

Department of Mathematics, Louisiana State University, Baton Rouge, LA 70803

E-mail address: cohen@math.lsu.edu
URL: http://math.lsu.edu/~cohen

Department of Mathematics, Northeastern University, Boston, MA 02115

E-mail address: alexsuciu@neu.edu
URL: http://www.math.neu.edu/~suciu

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